Curve Sketch: $y = x^4 - 3x^3 + 3x^2 - x$

x-Int:
$0 = x(x^3 - 3x^2 + 3x - 1)$

$\hookrightarrow x = 1: \; 1 - 3 + 3 - 1 = 0$
$\therefore (x-1)$ is a factor

y-Int: $(x=0)$
$y = 0$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 3 & -1 \\ & & -1 & 2 & -1 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$0 = x(x-1)(x^2 - 2x + 1)$
$0 = x(x-1)(x-1)^2$
$0 = x(x-1)^3$

$x = 0$ or $x = 1$
$(0,0)$ $(1,0)$

$y' = 4x^3 - 9x^2 + 6x - 1$

$\underline{x=1}: 4 - 9 + 6 - 1 = 0$
$\therefore (x-1)$ is factor
$y' = (x-1)^2(4x-1)$

$$\begin{array}{r|rrrr} -1 & 4 & -9 & 6 & -1 \\ & & -4 & 5 & -1 \\ \hline & 4 & -5 & 1 & 0 \end{array}$$

$(x-1)(4x^2 - 5x + 1)$ (4)
$4x^2 - 4x - x + 1$
$4x(x-1) - 1(x-1)$
$(x-1)(x-1)(4x-1)$

Critical Values:
$x = 1, \frac{1}{4}$

	$(x-1)^2$	$4x-1$	f'	f
$(-\infty, \frac{1}{4})$	+	−	−	Dec
$(\frac{1}{4}, 1)$	+	+	+	Inc
$(1, \infty)$	+	+	+	Inc

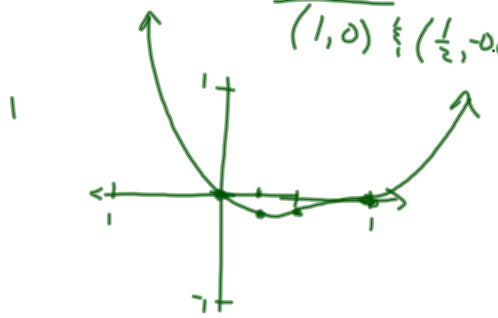
Local Max
None

Local Min
$(\frac{1}{4}, -0.1)$

$y'' = 12x^2 - 18x + 6$
$y'' = 6(2x^2 - 3x + 1)$
$2x^2 - 2x - 1x + 1$
$2x(x-1) - 1(x-1)$
$(x-1)(2x-1)$
$\boxed{y'' = 6(x-1)(2x-1)}$
$0 = 6(x-1)(2x-1)$
$x = 1, \frac{1}{2}$

	6	$x-1$	$2x-1$	f''	f
$(-\infty, \frac{1}{2})$	+	−	−	+	Up
$(\frac{1}{2}, 1)$	+	−	+	−	Down
$(1, \infty)$	+	+	+	+	Up

Inf. Points:
$(1, 0)$ & $(\frac{1}{2}, -0.06)$



Logarithmic Differentiation

A differentiation process that requires taking the logarithm of both sides before differentiating.

This process will be used in TWO circumstances:

I. Simplifying messy products and quotients

What would it involve to differentiate the following?

$$y = \frac{\left(x^2 - 1\right)^5 \sqrt{2x+9}\left(5x^3 + 2\right)^8}{(10x-1)\sqrt{5-x^7}}$$

- Quotient rule, multiple product rules and chain rules...

  This would be possible but it would be easier to differentiate a group of terms added and subtracted rather than multiplied and divided

Laws of logarithms will do exactly that...turn this mess into a addition and subtraction of terms.

$$y = \frac{\left(x^2 - 1\right)^5 \sqrt{2x+9}\left(5x^3 + 2\right)^8}{(10x-1)\sqrt{5-x^7}}$$

Example:

Differentiate: $y = \dfrac{\left(3-2x^{5}\right)^{6}\left(x^{5}-1\right)}{\left(2x+7\right)^{8}\left(x^{-5}+2\right)^{4}}$

$\ln y = 6\ln(3-2x^5) + \ln(x^5-1) - 8\ln(2x+7) - 4\ln(x^{-5}+2)$

$\cancel{y}\,\frac{1}{\cancel{y}}\frac{dy}{dx} = \left[6\left(\frac{-10x^4}{3-2x^5}\right) + \frac{5x^4}{x^5-1} - 8\left(\frac{2}{2x+7}\right) - 4\left(\frac{-5x^{-6}}{x^{-5}+2}\right)\right] y$

$\frac{dy}{dx} = \left[6\left(\frac{-10x^4}{3-2x^5}\right) + \frac{5x^4}{x^5-1} - 8\left(\frac{2}{2x+7}\right) - 4\left(\frac{-5x^{-6}}{x^{-5}+2}\right)\right]\left[\frac{\left(3-2x^{5}\right)^{6}\left(x^{5}-1\right)}{\left(2x+7\right)^{8}\left(x^{-5}+2\right)^{4}}\right]$

II. Base and exponent both variables

Have a look at this example:

$$y = x^{x^5}$$

- Does not fit either the power rule or the rules for an exponential function

...What can be done to help this crazy situation??

Of Course...take the logarithm of both sides!!

$$y = x^{x^5}$$

$$\ln y = \ln x^{x^5}$$

$$\ln y = \underbrace{x^5 \ln x}$$

$$(y)\left[\frac{1}{y}\frac{dy}{dx}\right] = \left[5x^4 \ln x + x^5\left(\frac{1}{x}\right)\right]y$$

$$\frac{dy}{dx} = \left(5x^4 \ln x + x^4\right)\left(x^{x^5}\right)$$

Example:

Differentiate: $y = \left(\ln x^5\right)^{\cos x}$

$$\ln y = \ln\left[\left(\ln x^5\right)^{\cos x}\right]$$

$$\ln y = \cos x\left[\ln\left(\ln x^5\right)\right]$$

$$(y)\frac{1}{y}\frac{dy}{dx} = \left[(-\sin x)\ln(\ln x^5) + \cos x\left(\frac{1}{\ln x^5}\cdot\frac{5x^4}{x^5}\right)\right]\left(\ln x^5\right)^{\cos x}$$

$$\ln\underbrace{\left(\ln x^7\right)}_{u}$$

$$\frac{1}{\ln x^7}\left(\frac{7x^6}{x^7}\right)$$

$$\ln\underbrace{\left(\ln\left(\ln\left(\tan x^2\right)\right)\right)}_{u}$$

$$\frac{1}{\ln(\ln(\tan x^2))}\left(\frac{1}{\ln(\tan x^2)}\right)\left(\frac{1}{\tan x^2}\right)\sec^2 x^2 (2x)$$

Practice Questions...

Page 395

#1, 2, 3