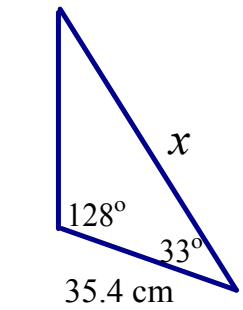
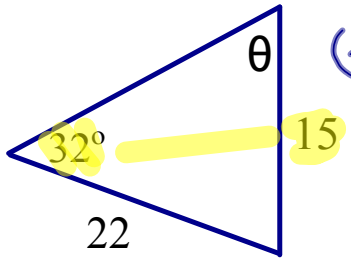
EXAMPLE #1 - Finding a side.



EXAMPLE #2 - Finding an angle.

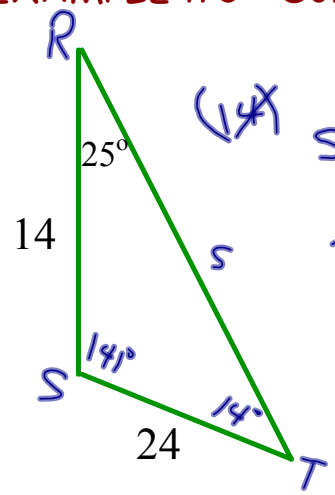


$$\frac{\sin\theta}{22} = \frac{\sin 32^\circ}{15}(22)$$

$$\sin\theta = \frac{\sin 32^\circ}{15}(22)$$

$$\theta = 51^\circ$$

EXAMPLE #3 - Solve the triangle.



$\frac{\sin T}{14} = \frac{\sin 25^\circ}{24}(14)$

$T = 14^\circ$ $\quad S = 141^\circ$

$\frac{S}{\sin 141^\circ} = \frac{24}{\sin 25^\circ}$

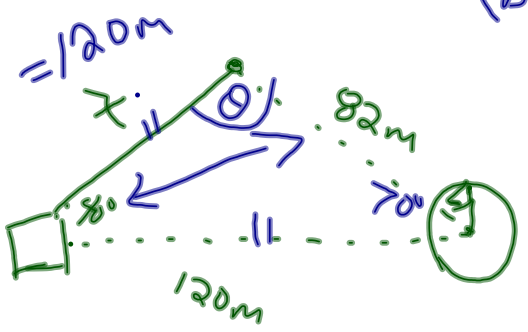
$S = \frac{24}{\sin 25^\circ}(\sin 141^\circ) = 35.7$

EXAMPLE #4 - Application

Suppose that Mr. Watters was playing a straight par-3 golf hole that was 120 m long. He hits one of his regular old slices that ends up 40° off line and is still 82 m from the hole.

(a) How far did his tee shot travel?

(b) If he somehow miraculously hits his next shot onto the green, what was the angle between the path of his first shot and the path followed by the second shot?



(b) $\frac{\sin\theta}{120} = \frac{\sin 40^\circ}{82}$

$\sin\theta = \frac{120\sin 40^\circ}{82}$

$\theta = 70^\circ$

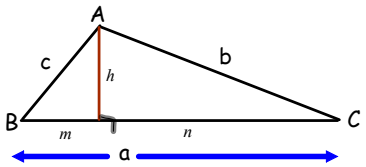
(a) $\frac{\sin 70^\circ}{120} = \frac{\sin 70^\circ}{x}$

$120\sin 70^\circ = x\sin 70^\circ$

$120 = x$

Law of Cosines

Derivation of the law of cosines...



$c^2 = h^2 + m^2$ ← $m = a - n$

$c^2 = h^2 + (a-n)^2$

$c^2 = h^2 + a^2 - 2an + n^2$

$c^2 = h^2 + n^2 + a^2 - 2an$ ← $h^2 + n^2 = b^2$

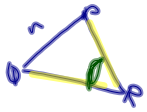
$c^2 = b^2 + a^2 - 2an$ ← $\cos C = \frac{n}{b}$, $n = b\cos C$

$c^2 = a^2 + b^2 - 2a(b\cos C)$

$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$

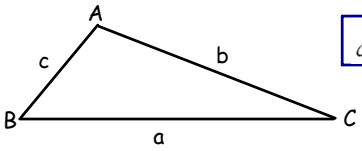
ΔPQR

$r^2 = p^2 + q^2 - 2pq\cos R$



Finding an unknown side...

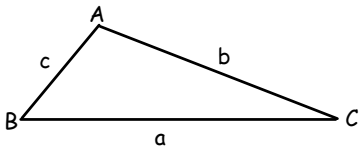
- 2 sides and a contained angle (SAS)



$$a^2 = b^2 + c^2 - 2bc\cos A$$

Finding an unknown angle...

- 3 known sides (SSS)



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

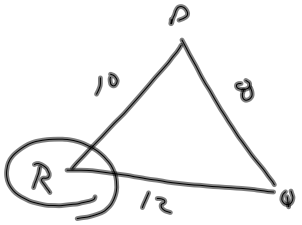
$a^2 = b^2 + c^2 - 2bc\cos A$

$\frac{a^2 - b^2 - c^2}{-2bc} = \frac{-2bc\cos A}{-2bc}$

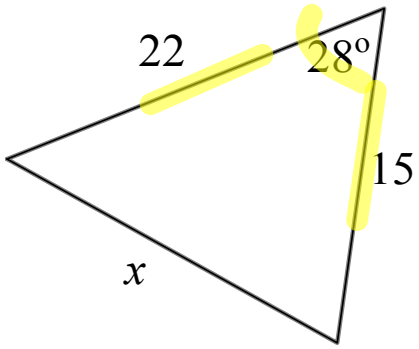
$\cos A = \frac{(a^2 - b^2 - c^2)}{2bc}$ ⇝ $-\frac{1}{3} = \frac{1}{-3}$

$= \frac{-a^2 + b^2 + c^2}{2bc}$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$



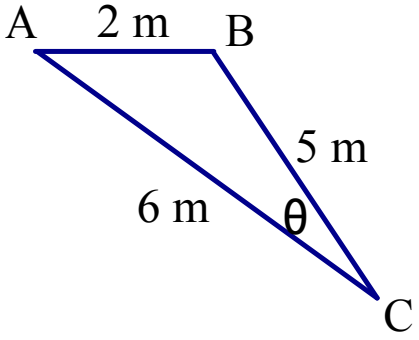
EXAMPLE: Finding an unknown side.



$x^2 = 22^2 + 15^2 - 2(22)(15)\cos 28°$

$x = 11.2$

EXAMPLE: Finding an unknown angle.



$\cos\theta = \dfrac{6^2 + 5^2 - 2^2}{2(6)(5)}$

EXAMPLE: Application question

A hockey net is 1.83m wide. A player shoots from a point where the puck is 13m from one goal post and 11.5m from the other. Within what angle must he make his shot to score?