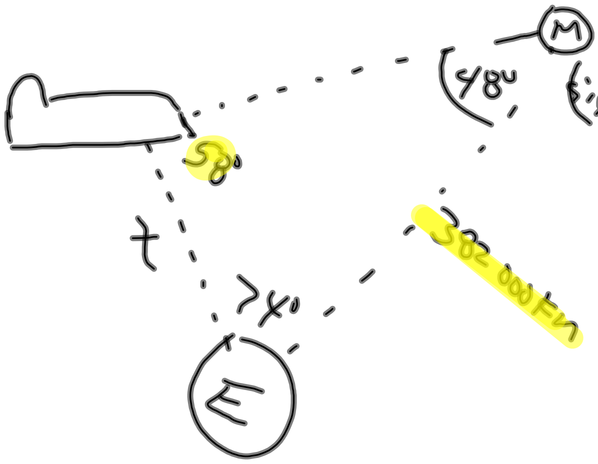
Warm Up

On a space flight, astronant Neil Armstrong reports that the angle formed by his lines of sight to the earth and to the moon was 58°. At the same time, the observer on the earth reports that the angle formed by her lines of sight to the spaceship and to the moon is 74°. If the moon is 382 000 km from the earth, how far is the spaceship from the tracking station?

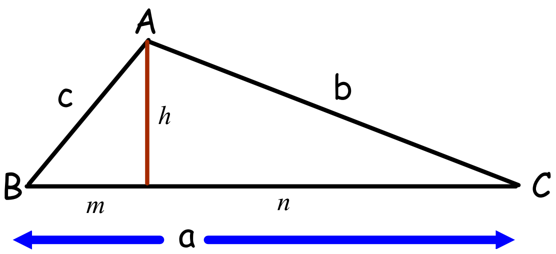


$$\cancel{(\sin 48)}\frac{x}{\cancel{\sin 48°}} = \frac{382\,000}{\sin 58°}(\sin 48°)$$

$$\underline{x = 334\,747 \text{ km}}$$

Law of Cosines

Derivation of the law of cosines...



$$c^2 = h^2 + m^2 \longleftarrow m = a - n$$

$$c^2 = h^2 + (a-n)^2$$

$$c^2 = h^2 + a^2 - 2an + n^2$$

$$c^2 = h^2 + n^2 + a^2 - 2an \longleftarrow h^2 + n^2 = b^2$$

$$c^2 = b^2 + a^2 - 2an \longleftarrow \cos C = \frac{n}{b}$$

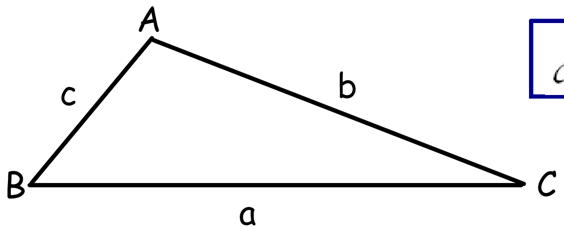
$$n = b\cos C$$

$$c^2 = a^2 + b^2 - 2a(b\cos C)$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

Finding an unknown side...

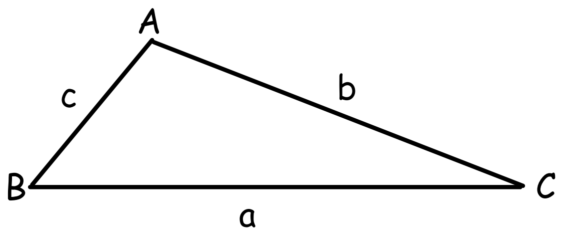
- 2 sides and a contained angle (SAS)



$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

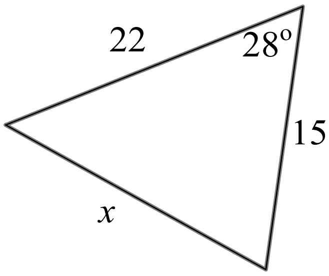
Finding an unknown angle...

- 3 known sides (SSS)

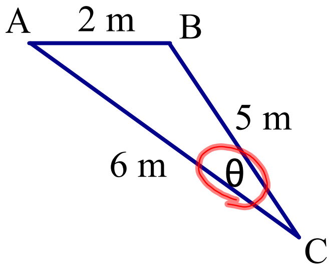


$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

EXAMPLE: Finding an unknown side.



EXAMPLE: Finding an unknown angle.



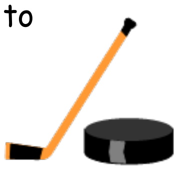
$$\cos\theta = \frac{5^2+6^2-2^2}{2(5)(6)}$$

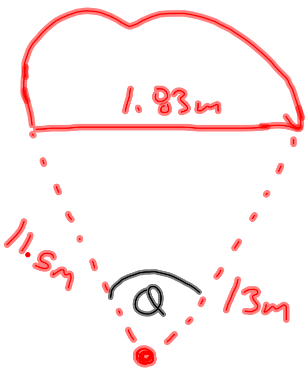
$\cos\theta =$ (5²+6²-2²)/(2(5)(6)) .95

$$\theta = 18^\circ$$

EXAMPLE: Application question

A hockey net is 1.83m wide. A player shoots from a point where the puck is 13m from one goal post and 11.5m from the other. Within what angle must he make his shot to score?





$$\cos\theta = \frac{11.5^2+13^2-1.83^2}{2(11.5)(13)}$$

$$\theta = 5^\circ$$

Homework...

Worksheet - Law of Cosines

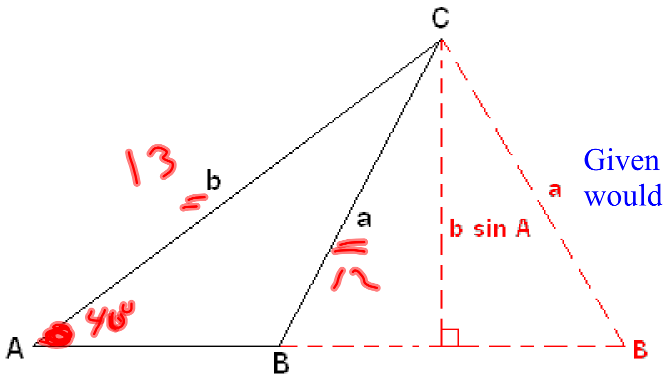
Left side:
#5, 6, 7

Right side:
#1 - 6

Solving Oblique Triangles: The Ambiguous Case

The ambiguous case

When using the law of sines to solve triangles, under special conditions there exists an ambiguous case where two separate triangles can be constructed (i.e., there are two different possible solutions to the triangle).



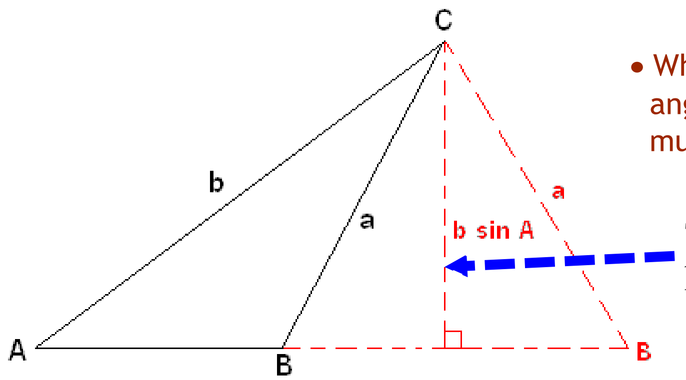
Given a general triangle ABC, the following conditions would need to be fulfilled for the case to be ambiguous:

- The only information known about the triangle is the angle A and the sides a and b, where the angle A is not the included angle of the two sides.
- The angle A is acute (i.e., $A < 90°$).
- The side a is shorter than the side b (i.e., $a < b$). (a is the altitude of a right triangle with angle A)
- The angle B is not a right angle (i.e., $a > b \sin A$).

Given all of the above premises are true, the angle B may be acute or obtuse; meaning, one of the following is true:

$$B = \arcsin \frac{b \sin A}{a} \quad \text{OR} \quad B = 180^\circ - \arcsin \frac{b \sin A}{a}$$

Summary: Ambiguous Case



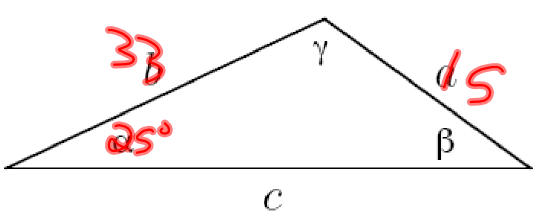
- When given two sides and a non-contained angle that is acute the ambiguous case must be examined

The altitude of the triangle will dictate how many solutions are possible.

$a < b \bullet \sin A$	No Solutions
$b \bullet \sin A < a < b$	Two Solutions
$b < a$	One Solution

Example 1:

Given that $\alpha = 25°$, $a = 15$, and $b = 33$, find the measure of angle β to the nearest degree. If there are two answers, give both of them. If there are no possible answers, write "none".



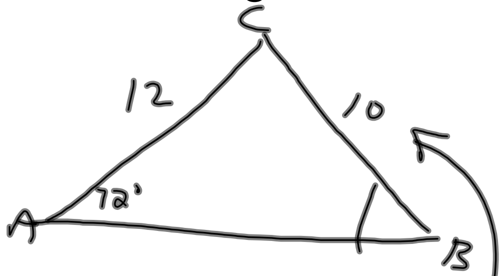
$b \sin A$
$= 33 \sin 25°$
$= 13.9$ $\therefore$ 2 Possible

$\frac{\sin B}{33} = \frac{\sin 25°}{15}$

$B = 68°$ OR $B = 180° - 68° = 112°$

Example 2:

Solve the triangle ABC if $a = 10$, $b = 12$ and angle A = 72°.



$12 \sin 72°$
$= 11.4$

$11.4 > 10$ $\therefore$ Not possible

$\frac{\sin B}{12} = \frac{\sin 72°}{10}$

$\sin B = 1.1413$

$\varnothing$

Attachments

Worksheet - Law of Cosines.doc