Matrix Multiplication

In order to multiply matrices, the number of columns in the 1st matrix must equal the number of rows in the 2nd matrix.

Product Dimensions:

(# rows 1st) x (# columns 2nd)

Always multiply a row through a column, adding the products as you go.

Ex.

$$\begin{pmatrix} 2 & 7 \\ 3 & 5 \end{pmatrix} \text{ x } \begin{pmatrix} 6 & -2 & 0 & -1 \\ 7 & 1 & 5 & 4 \end{pmatrix} = \begin{bmatrix} (12+49) & (-4+7) & (0+35) & (-2+28) \\ (18+35) & (-6+5) & (0+25) & (-3+20) \end{bmatrix}$$

(2 x 2) (2 x 4)

$$= \begin{bmatrix} 61 & 3 & 35 & 26 \\ 53 & -1 & 25 & 17 \end{bmatrix}$$

(2 x 4)

Product: 2x4

(3x7)x(7x5) Product is 3x5

Your turn! Given...

```
[A]
      [[5  1  -1]
       [6  2  4 ]]
```

```
[B]
      [[4   -1]
       [2   -5]
       [-3  0 ]]
```

(2x3) (3x2)

$$\begin{pmatrix} 5 & 1 & -1 \\ 6 & 2 & 4 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & -5 \\ -3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 20+2+3 & -5-5+0 \\ 24+4-12 & -6-10+0 \end{pmatrix}$$

$$\begin{pmatrix} 25 & -10 \\ 16 & -16 \end{pmatrix}$$

```
[A][B]
      [[25  -10]
       [16  -16]]
```

Exercises

1. Evaluate the following.

a) $\underset{2\times 2}{\overset{A}{\begin{pmatrix} -3 & 2 \\ 3 & 11 \end{pmatrix}}} \underset{3\times 1}{\overset{B}{\begin{pmatrix} 3 \\ -1 \end{pmatrix}}}$, b) $\underset{(2\times 2)}{\begin{pmatrix} 4 & 2 \\ 5 & 11 \end{pmatrix}} \underset{(2\times 2)}{\begin{pmatrix} 3 & 10 \\ -1 & 9 \end{pmatrix}}$, c) $\underset{(2\times 2)}{\begin{pmatrix} 2 & 1 \\ 1 & 9 \end{pmatrix}} \underset{(2\times 3)}{\begin{pmatrix} 2 & 0 & 2 \\ 5 & 13 & 1 \end{pmatrix}}$.

Answers

1. a) $\begin{pmatrix} -11 \\ -2 \end{pmatrix}$, b) $\begin{pmatrix} 10 & 58 \\ 4 & 149 \end{pmatrix}$, c) $\begin{pmatrix} 9 & 13 & 5 \\ 47 & 117 & 11 \end{pmatrix}$.

a) $\begin{pmatrix} -9-2 \\ 9-11 \end{pmatrix} = \begin{pmatrix} -11 \\ -2 \end{pmatrix}$

b) $\begin{pmatrix} 12-2 & 40+18 \\ 15-11 & 50+99 \end{pmatrix} = \begin{pmatrix} 10 & 58 \\ 4 & 149 \end{pmatrix}$

c) $\begin{pmatrix} 4+5 & 0+13 & 4+1 \\ 2+45 & 0+117 & 2+9 \end{pmatrix} = \begin{pmatrix} 9 & 13 & 5 \\ 47 & 117 & 11 \end{pmatrix}$

Warm Up

Given the following matrices...

$$X = \begin{pmatrix} -1 & -2 \\ 3 & 1 \\ 2 & 5 \end{pmatrix} \qquad Y = \begin{pmatrix} 4 & -2 & 6 \\ 1 & 3 & 0 \end{pmatrix}$$

3×2 $\qquad$ 2×3

$$Z = \begin{pmatrix} 1 & -5 & 6 \\ -3 & 0 & 2 \\ 2 & -1 & 7 \end{pmatrix}$$

Determine the value of $2XY - 5Z$

Check your work on a TI-83 calculator when finished

$$2\begin{pmatrix} -1 & -2 \\ 3 & 1 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 4 & -2 & 6 \\ 1 & 3 & 0 \end{pmatrix} - 5\begin{pmatrix} 1 & -5 & 6 \\ -3 & 0 & 2 \\ 2 & -1 & 7 \end{pmatrix}$$

$$2\begin{pmatrix} -4-2 & 2-6 & -6+0 \\ 12+1 & -6+3 & 18+0 \\ 8+5 & -4+15 & 12+0 \end{pmatrix} - \begin{pmatrix} 5 & -25 & 30 \\ -15 & 0 & 10 \\ 10 & -5 & 35 \end{pmatrix}$$

$$2\begin{pmatrix} -6 & -4 & -6 \\ 13 & -3 & 18 \\ 13 & 11 & 12 \end{pmatrix} - \begin{pmatrix} 5 & -25 & 30 \\ -15 & 0 & 10 \\ 10 & -5 & 35 \end{pmatrix}$$

$$\begin{pmatrix} -12 & -8 & -12 \\ 26 & -6 & 36 \\ 26 & 22 & 24 \end{pmatrix} - \begin{pmatrix} 5 & -25 & 30 \\ -15 & 0 & 10 \\ 10 & -5 & 35 \end{pmatrix}$$

$$= \begin{pmatrix} -17 & 17 & -42 \\ 41 & -6 & 26 \\ 16 & 27 & -11 \end{pmatrix}$$

```
2*[A]*[B]-5[C]
  [[-17 17  -42]
   [41   -6 26 ]
   [16   27  -11]]
```

SPECIAL MATRICES

(1) Zero Matrix - a.k.a "null matrix"

- all entries are 0's. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(2) Identity Matrix - denoted by "I"; a.k.a "unit matrix"

The Identity Matrix, called I, is a square matrix with all elements 0 except the *principal diagonal* which has all ones:

Major Diagonal

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2 × 2 identity matrix

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3 × 3 identity matrix

$$\boxed{A \cdot I = I \cdot A = A}$$

EXAMPLE... $\begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1+0 & 0-3 \\ 2+0 & 0+1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix}$$

Ex. Find AB & BA

$$\underset{\textbf{A}}{\begin{pmatrix} 1 & 4 \\ 2 & 9 \end{pmatrix}} \quad \underset{\textbf{B}}{\begin{pmatrix} 9 & -4 \\ -2 & 1 \end{pmatrix}}$$

$$\begin{pmatrix} 9+-8 & -4+4 \\ 18-18 & -8+9 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Identity Matrix

Finding the Inverse of a Matrix

Ex. $\left(\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 2 & 9 & 0 & 1 \end{array}\right)$

Perform elementary operations to make left side unit matrix

Using Matrices to solve a system of equations...

Example: Solve... $x - 2y = 2$ & $3x + 4y = 16$

$x - 2y = 2$
$3x + 4y = 16$

$$\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 16 \end{pmatrix}$$

- Construct an augmented matrix...

$$\begin{array}{c} ① \\ ② \end{array}\left(\begin{array}{cc|c} 1 & -2 & 2 \\ 3 & 4 & 16 \end{array}\right)$$

Elementary Row Operations

I. Interchange two rows.

II. Multiply one row by a nonzero number.

III. Add a multiple of one row to a different row.

- Perform elementary row operations to put augmented matrix into row-echelon form...

Row-Echelon Form

A matrix is said to be in **row-echelon** form if

1. All rows consisting entirely of zeros are at the bottom.
2. In each row, the first non-zero entry from the left is a 1, called the **leading 1**.
3. The leading 1 in each row is to the right of all leading 1's in the rows above it.

If, in addition, each leading 1 is the only non-zero entry in its column, then the matrix is in **reduced row-echelon form**.

$$\left(\begin{array}{cc|c} 1 & -2 & 2 \\ 3 & 4 & 16 \end{array}\right) \xrightarrow{②-3①} \begin{array}{ccc} x & y & \\ \end{array}$$
$$\left(\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 10 & 10 \end{array}\right)$$

Row to be changed

$\cancel{0x} + 10y = 10$
$y = 1$

$x - 2(1) = 2$
$x = 4$

$\boxed{(4,1)}$

Examples:

1) $\begin{cases} 2x + y = 2 \\ 4x + 2y = -5 \end{cases}$

2) $\begin{cases} 5x = 2y - 7 \\ 3x + 4y = 1 \end{cases}$

3) $\begin{array}{l} -3x + 3y = 4 \\ -x + y = 3 \end{array}$

4) $\begin{array}{rcl} x - 2y & = & -1 \\ -2x + 4y & = & 2 \end{array}$