Examples:

1) $\begin{cases} 2x + y = 2 \\ 4x + 2y = -5 \end{cases}$

2) $\begin{cases} 5x = 2y - 7 \\ 3x + 4y = 1 \end{cases}$

$$\left(\begin{array}{cc|c} 5 & -2 & -7 \\ 3 & 4 & 1 \end{array}\right)$$

$$\xrightarrow[5②]{3①} \left(\begin{array}{cc|c} 15 & -6 & -21 \\ 15 & 20 & 5 \end{array}\right)$$

$$\xrightarrow{②-①} \left(\begin{array}{cc|c} 15 & -6 & -21 \\ 0 & 26 & 26 \end{array}\right)$$

$$\frac{26y}{26} = \frac{26}{26}$$

$$y = 1$$

$$5x - 2(1) = -7$$

$$5x = -7 + 2$$

$$\frac{5x}{5} = \frac{-5}{5} \quad x = -1$$

OR

$$\xrightarrow{5②-3①} \left(\begin{array}{cc|c} 5 & -2 & -7 \\ 0 & 26 & 26 \end{array}\right)$$

$$26y = 26$$

$$y = 1$$

3) $-3x + 3y = 4$
$-x + y = 3$

4) $\begin{aligned} x - 2y &= -1 \\ -2x + 4y &= 2 \end{aligned}$

$$\left(\begin{array}{cc|c} 1 & -2 & -1 \\ -2 & 4 & 2 \end{array}\right)$$

$$\xrightarrow{②+2①} \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 0 & 0 \end{array}\right)$$

$\therefore$ dependent, infinite solutions

Let $y = t$

$$x - 2t = -1$$

$$x = -1 + 2t$$

$$(-1 + 2t, t)$$

$$(-1, 0), (1, 1) \quad (199, 100)$$

(3) Inverses

- two matrices whose product is a unit matrix are called inverses.
- the inverse matrix is denoted A^{-1}.
- we will look at the inverses of 2 by 2 matrices.
- otherwise, we can use the TI-83 to get the inverse matrix.
- if det A = 0,
  then there is NO inverse and it is called a **singular matrix**.

$$\boxed{A \cdot A^{-1} = A^{-1} \cdot A = I}$$

$$\boxed{\textit{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \textit{then } A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

A *determinant* is a square array of numbers (written within a pair of vertical lines) which represents a certain sum of products. It produces a single number (a *scalar* quantity).

In general, we find the value of a 2×2 determinant as follows:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \text{Det} = ad - bc$$

Det A = (product of major diagonal) - (product of other diagonal)

$$\boxed{\text{Det A} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb}$$

Example: Determinant

$$A = \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} = 4 \times 3 - 2 \times 1$$

$$= 12 - 2$$

$$= 10$$

$$A^{-1} = \frac{1}{10}\begin{pmatrix} 3 & -1 \\ -2 & 4 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{2}{10} & \frac{4}{10} \end{pmatrix}$$

Example: Find the inverse, A^{-1}, of

$$A = \begin{pmatrix} 2 & -3 \\ 4 & -7 \end{pmatrix},$$

det = -14 - (-12)
= -2

$$A = -14 - (-12) = -2$$

$$A^{-1} = \frac{1}{-2}\begin{pmatrix} -7 & 3 \\ -4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3.5 & -1.5 \\ 2 & -1 \end{pmatrix}$$

Your turn...

Ex.

$$A = \begin{pmatrix} 0 & -3 \\ 4 & 5 \end{pmatrix} \qquad A^{-1} =$$

Check with the TI-83...

det = 0 - (-12)
= 12

$$\frac{1}{12}\begin{pmatrix} 5 & 3 \\ -4 & 0 \end{pmatrix} = \begin{pmatrix} \frac{5}{12} & \frac{1}{4} \\ -\frac{1}{3} & 0 \end{pmatrix}$$

Inverses

$$\left(\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & -7 & 0 & 1 \end{array}\right)$$

$\tfrac{1}{2}$①

$$\left(\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 4 & -7 & 0 & 1 \end{array}\right) \xrightarrow{②-4①} \left(\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & -1 & -2 & 1 \end{array}\right)$$

$$\xrightarrow{② \times -1} \left(\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 2 & -1 \end{array}\right)$$

$$\xrightarrow{① + \frac{3}{2}②} \left(\begin{array}{cc|cc} 1 & 0 & \frac{7}{2} & -\frac{3}{2} \\ 0 & 1 & 2 & -1 \end{array}\right)$$

Inverse

SOLVING LINEAR SYSTEMS: INVERSE MATRIX METHOD

The matrix form associated with the coefficients, variables and constants of the linear system

$$\begin{aligned} 5x + 2y &= 14 \\ 3x - 7y &= 33 \end{aligned}$$

Must be in this form

can be written as

A^{-1} A 2x1 A^{-1} 2x2

$$\begin{pmatrix} 5 & 2 \\ 3 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 33 \end{pmatrix}.$$

This matrix is called the **constant matrix**.

2x2

the **coefficient matrix**, C

is matrix is the **variable matrix**, V

Example Solve using the inverse matrix method.

$$\begin{aligned} 5x + 2y &= 14 \\ 3x - 7y &= 33 \end{aligned}$$

Solution Write the system in the form of a matrix equation.

$$\underset{\uparrow\; C}{\begin{pmatrix} 5 & 2 \\ 3 & -7 \end{pmatrix}}\underset{\uparrow\; V}{\begin{pmatrix} x \\ y \end{pmatrix}} = \begin{pmatrix} 14 \\ 33 \end{pmatrix}$$

Find the inverse matrix, C^{-1}

$$\begin{aligned} \det C &= ad - bc \\ &= (5)(-7) - (2)(3) \\ &= -41 \end{aligned}$$

$$\begin{aligned} C^{-1} &= \frac{1}{\det C}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \\ &= \frac{1}{-41}\begin{pmatrix} -7 & -2 \\ -3 & 5 \end{pmatrix} \\ &= \begin{pmatrix} \frac{7}{41} & \frac{2}{41} \\ \frac{3}{41} & \frac{-5}{41} \end{pmatrix} \end{aligned}$$

Solve the matrix equation. Multiply both sides of the matrix equation by C^{-1}.

$$\begin{pmatrix} \frac{7}{41} & \frac{2}{41} \\ \frac{3}{41} & -\frac{5}{41} \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 3 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{7}{41} & \frac{2}{41} \\ \frac{3}{41} & -\frac{5}{41} \end{pmatrix}\begin{pmatrix} 14 \\ 33 \end{pmatrix}$$

2x2 2x2 2x2 2x1

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{98+66}{41} \\ \frac{42-165}{41} \end{pmatrix}$$

2x2 2x1

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{164}{41} \\ -\frac{123}{41} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

Verify the answer in the original equation.

Thus, from the matrix form $x = 4$, $y = -3$.

Solving a System Using Identity Matrix

Solve:

$$7x - 2y = 3$$
$$11x - 3y = 5$$

1. Set up Coefficient Matrix

matrix formed by coefficients in a linear system of equations

$$\begin{pmatrix} 7 & -2 \\ 11 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

2. Find inverse of . . .

$$\det = -21 - (-22) = 1$$

$$\text{Inverse} = \frac{1}{1}\begin{pmatrix} -3 & 2 \\ -11 & 7 \end{pmatrix}$$

3. Multiply Both Sides of Equations by Inverse

$$\begin{pmatrix} -3 & 2 \\ -11 & 7 \end{pmatrix}\begin{pmatrix} 7 & -2 \\ 11 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ -11 & 7 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -9+10 \\ -33+35 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$x = 1$$
$$y = 2$$

Homework

- Attempt any 4 from sheet using inverse matrices