Derivatives

*The concept of a **Derivative** is at the core of Calculus and modern mathematics. The definition of the derivative can be approached in two different ways. One is geometrical (as a slope of a curve) and the other one is physical (as a rate of change).*

$\boxed{2}$ **Definition** The **derivative of a function f at a number a**, denoted by $f'(a)$, is

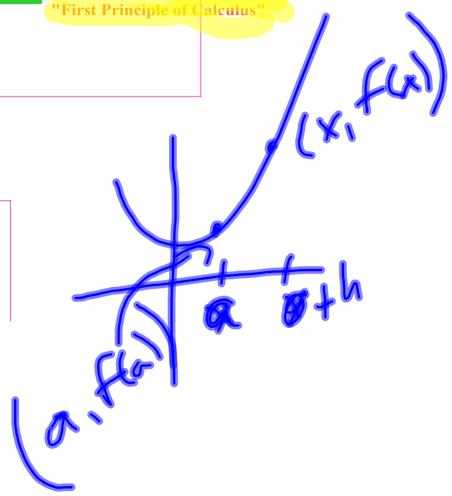
$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

Also known as the "First Principle of Calculus"

if this limit exists.

...or this definition can also be expressed as

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$



Most common notations used to represent the derivative…

$$f'(x) \quad \& \quad \frac{dy}{dx}$$

Examples:

Use the definition of a derivative to differentiate each of the following:

1. $y = x^2 - 5x + 2$

$f(x) = x^2 - 5x + 2$

$f(x+h) = (x+h)^2 - 5(x+h) + 2$

$= x^2 + 2xh + h^2 - 5x - 5h + 2$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{(\cancel{x^2} + 2xh + h^2 \cancel{-5x} - 5h \cancel{+2}) - (\cancel{x^2} \cancel{-5x} \cancel{+2})}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 - 5h}{h} = \frac{0}{0}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + h - 5)}{\cancel{h}}$$

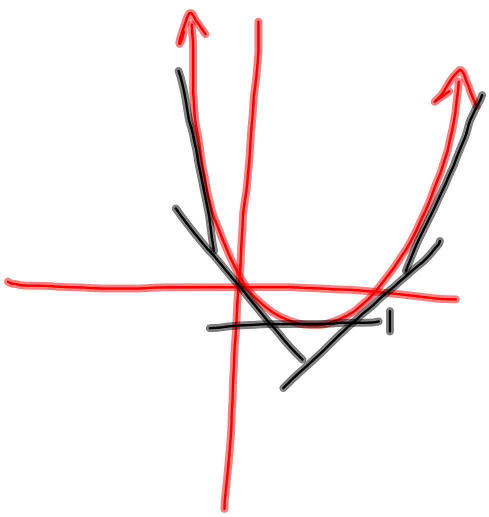
Derivative

→ Slope

→ Rate of change

$$\boxed{f'(x) = 2x - 5}$$

*Slope of a tangent at any x-value



2. $f(x)=\sqrt{2x-5}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f(x+h)=\sqrt{2(x+h)-5}$$

$$f(x+h)=\sqrt{2x+2h-5}$$

$$\lim_{h\to 0}\frac{\sqrt{2x+2h-5}-\sqrt{2x-5}}{h}\left(\frac{\sqrt{2x+2h-5}+\sqrt{2x-5}}{\sqrt{2x+2h-5}+\sqrt{2x-5}}\right)$$

$$\lim_{h\to 0}\frac{(\cancel{2x}+2h\cancel{-5})-(\cancel{2x}\cancel{-5})}{h\left(\sqrt{2x+2h-5}+\sqrt{2x-5}\right)}$$

$$\lim_{h\to 0}\frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2x+2h-5}+\sqrt{2x-5}\right)}$$

$2h \to 0$

$\sqrt{x}+\sqrt{x}$
$=2\sqrt{x}$

$$f'(x)=\frac{2}{\sqrt{2x-5}+\sqrt{2x-5}}$$

$$f'(x)=\frac{1\cancel{2}}{\cancel{2}\sqrt{2x-5}}=\frac{1}{\sqrt{2x-5}}$$

3. Determine $f'(-2)$ given that $f(x)=\dfrac{x^2-1}{2x+3}$

$$f(x+h)=\frac{(x+h)^2-1}{2(x+h)+3}$$

$$f(x+h)=\frac{x^2+2xh+h^2-1}{2x+2h+3}$$

$$\lim_{h\to 0}\frac{\dfrac{x^2+2xh+h^2-1}{2x+2h+3}-\dfrac{x^2-1}{2x+3}}{h}$$

$$\lim_{h\to 0}\left[\frac{(x^2+2xh+h^2-1)(2x+3)-(x^2-1)(2x+2h+3)}{(2x+2h+3)(2x+3)}\right]\times\frac{1}{h}$$

$$\lim_{h\to 0}\frac{(\cancel{2x^3}+\cancel{3x^2}+4x^2h+6xh+2xh^2+3h^2-\cancel{2x}-\cancel{3})-(\cancel{2x^3}+2x^2h+\cancel{3x^2}-\cancel{2x}-2h-\cancel{3})}{(2x+2h+3)(2x+3)}\cdot\frac{1}{h}$$

$$\lim_{h\to 0}\frac{(2x^2h+6xh+2xh^2+3h^2+2h)}{(2x+2h+3)(2x+3)\,h}$$

$$\lim_{h\to 0}\frac{\cancel{h}(2x^2+6x\underbrace{+2xh}_{\to 0}\underbrace{+3h}_{\to 0}+2)}{(2x\underbrace{+2h}_{\to 0}+3)(2x+3)\cancel{(h)}}$$

$$f'(x)=\frac{2x^2+6x+2}{(2x+3)(2x+3)}$$

$$f'(x)=\frac{2x^2+6x+2}{(2x+3)^2}$$

$$f'(-2)=\frac{2(-2)^2+6(-2)+2}{(2(-2)+3)^2}$$

$$=\frac{8+-12+2}{1}$$

$$=-2$$

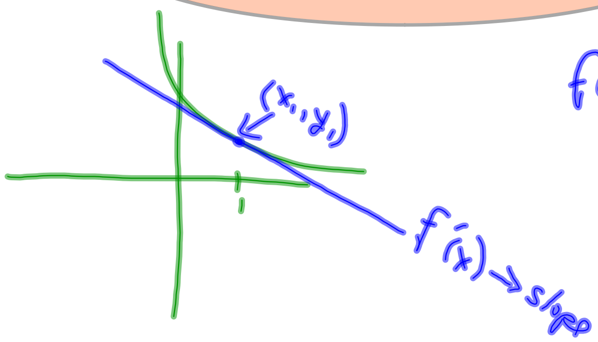
Example:

Determine the equation of a tangent drawn to the curve $f(x)=\dfrac{2}{1-3x}$ at $x=1$.

$y=6x-7$

Remember that the equation of a line is found by using the point-slope formula… $y-y_1=m(x-x_1)$

$f'(x)=\dfrac{6}{(1-3x)^2}$



$$f(x+h)=\frac{2}{1-3(x+h)}=\frac{2}{1-3x-3h}$$

$$\lim_{h\to 0}\frac{\frac{2}{1-3x-3h}-\frac{2}{1-3x}}{h}$$

$$\lim_{h\to 0}\frac{2(1-3x)-2(1-3x-3h)}{(1-3x-3h)(1-3x)}\cdot\frac{1}{h}$$

$$\lim_{h\to 0}\frac{\overset{6\cancel{h}}{\cancel{2-6x-2+6x+6h}}}{(1-3x-\underset{0}{3h})(1-3x)}\cdot\frac{1}{\cancel{h}}$$

$$f'(x)=\frac{6}{(1-3x)^2}$$

At $x=1$:

① $f(1)=\dfrac{2}{1-3(1)}=\dfrac{2}{-2}=-1$

$(1,-1) \leftarrow$ Point

② Slope:

$f'(-1)=\dfrac{6}{(1-3(1))^2}=\dfrac{6}{4}=\dfrac{3}{2}$

Tangent: $\longrightarrow$

$$y+1=\frac{3}{2}(x-1)$$
$$2y+2=3(x-1)$$
$$2y+2=3x-3$$
$$0=3x-2y-5 \quad \text{or} \quad y=\frac{3}{2}x-\frac{5}{2}$$

Practice:

Page 76 & 77

#4, 5, 8, 10, 11, 12

Attachments

Review of antiderivatives, area and volume.doc