Warm Up

Evaluate each of the following if they exist...

$$\lim_{x\to\infty}\frac{\left(2-3x^2\right)^2}{\left(2x^2+1\right)\left(3x^2-5\right)}$$

$$\lim_{x\to\infty}\frac{\frac{4}{x^4}-\frac{12x^2}{x^4}+\frac{9x^4}{x^4}}{\frac{6x^4}{x^4}-\frac{7x^2}{x^4}-\frac{5}{x^4}}$$

$$=\frac{0-0+9}{6-0-0}$$

$$=\frac{9}{6}=\frac{3}{2}$$

$$\lim_{x\to-5^+}\frac{|x+5|}{4x+20}$$

$-4.999\ldots$

$$\lim_{x\to-5^+}\frac{|x+5|}{4(x+5)}$$

$$=\frac{|-4.999\ldots+5|}{4(-4.999\ldots+5)}$$

$$=\frac{|0.0000\ldots1|}{4(0.0000\ldots1)}$$

$$\lim_{x\to2}\frac{\sqrt{x}-\sqrt{2}}{x^4-16}$$

$$\lim_{x\to2}\frac{\sqrt{x}-\sqrt{2}}{(x^2-4)(x^2+4)}$$

$$\lim_{x\to2}\frac{\sqrt{x}-\sqrt{2}}{(x-2)(x+2)(x^2+4)}\left(\frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}\right)$$

$$\lim_{x\to2}\frac{\cancel{\sqrt{x}-\sqrt{2}}}{\cancel{(\sqrt{x}-\sqrt{2})}(\sqrt{x}+\sqrt{2})(x+2)(x^2+4)}$$

$$=\frac{1}{(\sqrt{2}+\sqrt{2})(4)(8)}$$

$$=\frac{1}{64\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$=\frac{\sqrt{2}}{128}$$

• Exponential Functions

$\lim_{x\to\infty} e^x$

$= \to \infty$

DNE

$\lim_{x\to-\infty} e^x = 0$

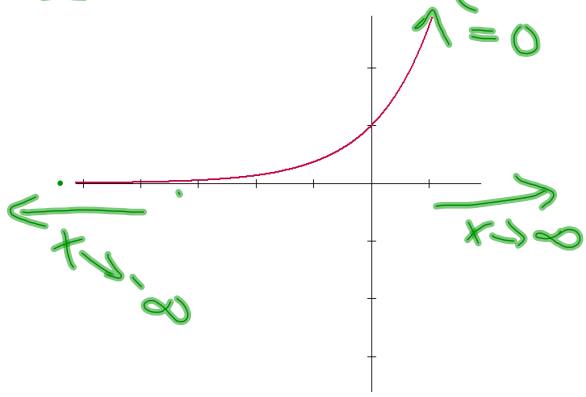
$= e^{-\infty}$

$= \frac{1}{e^{\infty}}$

$= 0$

e = Euler's Number

$= 2.71\ldots$



Try each of these...

1. $\lim_{x\to-\infty} 2^{3-x}$

$= 2^{3-(-\infty)}$

$= 2^{3+\infty}$

$\to \infty$

DNE

2. $\lim_{x\to\infty} \frac{2^x}{5^x}$

$= \left(\frac{2}{5}\right)^x$

$(<1)^{\infty}$

$= 0$

3. $\lim_{x\to\infty} \frac{3^x}{4}$

$= \frac{3^{\infty}}{4}$

$\to \infty$

DNE

4. $\lim_{x\to-\infty} \frac{1}{5^x}$

$\frac{1}{5^{-\infty}}$

$= 5^{\infty}$

$\to \infty$

DNE

Homework:

limits at infinity

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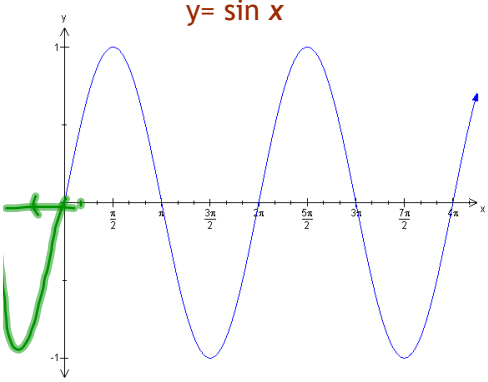
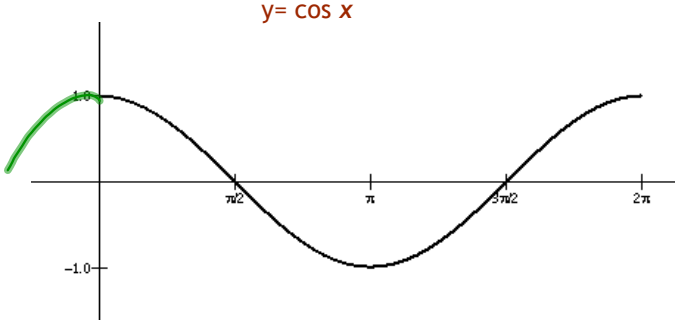
Limits of Trigonometric Functions

y= sin *x*



$$\lim_{x \to 0} \sin x = 0$$

y= cos *x*



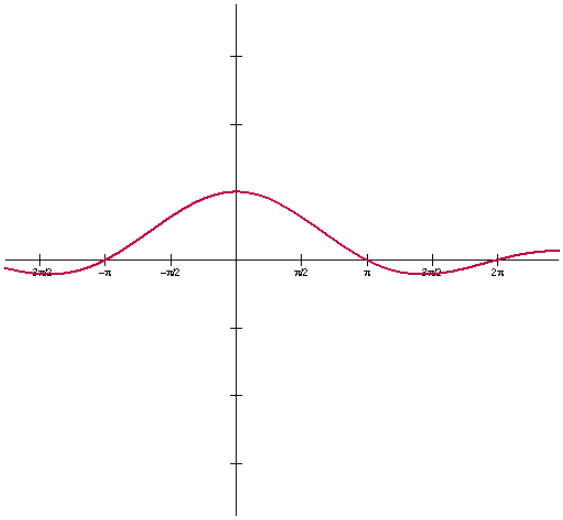
$$\lim_{x \to 0} \cos x = 1$$

The most important limit involving trigonometric functions is...

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

Must be in Radians!

Let's examine this limit by first examining the graph of $f(\theta) = \frac{\sin\theta}{\theta}$



Here is a look at the table...

X	Y_1
-.03	.99985
-.02	.99993
-.01	.99998
0	ERROR
.01	.99998
.02	.99993
.03	.99985

X=-.03

IDENTITY: $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

or $\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$

1

Examples:

How will this identity be used?

$$\lim_{x\to 0}\frac{\sin(5x)}{x}$$

argument

$$\lim_{x\to 0} 5\left(\frac{\sin 5x}{5x}\right)$$

$$= 5(1)$$

$$= 5$$

$$\lim_{x\to 0}\frac{8x}{\sin(5x)}$$

$$\lim_{x\to 0}\frac{8(5x)}{(5)\sin(5x)}$$

$$= \frac{8}{5}(1)$$

$$= \frac{8}{5}$$

$$\lim_{x\to \frac{\pi}{4}}\frac{4x}{\sin x}$$

$$= \frac{\left(\cancel{4}\frac{\pi}{\cancel{4}}\right)}{\sin\frac{\pi}{4}}$$

$$= \frac{\pi}{\left(\frac{1}{\sqrt{2}}\right)} = \pi\sqrt{2}$$

$$\lim_{x\to 0}\frac{6x}{\cos 3x}$$

$$= \frac{6(0)}{\cos 0}$$

$$= \frac{0}{1}$$

$$= 0$$

$$\lim_{x\to 0}\frac{\sin 2x}{\tan 7x}$$

$$\lim_{x\to 0}\frac{\sin^3 2x}{5x^3+10x^4}$$

$$\lim_{x\to 0} \sin 2x \cot 7x$$

$$\lim_{x\to 0} \sin 2x \frac{\cos 7x}{\sin 7x}$$

$$\lim_{\theta\to 0}\frac{\theta}{\sin\theta} = 1$$

$$\lim_{x\to 0}\left(\frac{\sin(2x)}{2x}\right)\left(\frac{7x}{\sin 7x}\right)\cos 7x\left(\frac{2\cancel{x}}{7\cancel{x}}\right)$$

$$= (1)(1)(1)\left(\frac{2}{7}\right)$$

$$= \frac{2}{7}$$

Attachments

qformula.wma