Warm Up

Differentiate:

$$f(x)=\sqrt[4]{\frac{(x^5-1)^{-2}+3x^7}{x\sqrt{3x-5}}}$$

$$y=\left(x^4+5x\sqrt{x+\sqrt[3]{x^3+8}}\right)^4$$

Holistic Marking:

→ Transfer & Minor Calculation errors

→ Conceptual errors:

a) $f'(x) = 42x^5 + 10x^{-6} - 40x^{-8}$ (4)

b) $f'(x) = \dfrac{(21x^2-1)(3x^4-x^2-5) - (7x^3-x)(12x^3-2x)}{(3x^4-x^2-5)^2}$ (4)

c) $y' = 4(3\sqrt{x}+x^3)^3\left(\frac{3}{2}x^{-\frac{1}{2}}+3x^2\right)$ (4)

d) $h'(x) = (2x^{-3}+\frac{1}{5}x^{-\frac{4}{5}})(4x^5-x^3+6x) + (8-2x^{-2}+\sqrt[5]{x})(20x^4-3x^2+6)$ (4)

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a) $f'(x) = \dfrac{\left[(-40x^{-6})(4x^3+1)^5 + 8x^{-5}\left[5(4x^3+1)^4(12x^2)\right]\right]\sqrt[4]{6-x^3} - \left[8x^{-5}(4x^3+1)^5\right]\left[\frac{1}{4}(6-x^3)^{-\frac{3}{4}}(-3x^2)\right]}{(\sqrt[4]{6-x^3})^2}$ (5)

$(\sqrt{6-x^3})$

$5x^{3/8}$

$\frac{5}{8}(x^3)^{-3/8}(3x^2)$

b) $y' = \frac{21}{2}x^6 - \frac{1}{2}(6x^8)^{-\frac{1}{2}}(48x^7) + \frac{15}{8}x^{-\frac{5}{8}} + 2x^{-2}$ (5)

c) $g'(x) = \left[6(4x^2-x^{-3})^5(8x+3x^{-4})\right](2x^2-x+5)^{-4}\sqrt{9-25x^2} + \left[-4(2x^2-x+5)^{-5}(4x-1)\right](4x^2-x^{-3})^6\sqrt{9-25x^2} +$

$\left[\frac{1}{2}(9-25x^2)^{-\frac{1}{2}}(-50x)\right](4x^2-x^{-3})^6(2x^2-x+5)^{-4}$ (5)

d) $y' = \frac{1}{3}\left[8x^6-(5x)^{1/2}+(3x^2-2x)^{1/4}\right]^{-2/3}\left[48x^5-\frac{1}{2}(5x)^{-\frac{1}{2}}(5) + \frac{1}{4}(3x^2-2x)^{-3/4}(6x-2)\right]$ (5)

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a) $= \frac{1}{2}\left[\dfrac{8x^5-x^6(2-x^{-3})^{1/2}}{(3x^2+27)^{1/3}(x^7+1)^8}\right]^{-\frac{1}{2}} \cdot \left[\dfrac{\left\{40x^4-\left[6x^5(2-x^{-3})^{1/2}+x^6\left[\frac{1}{2}(2-x^{-3})^{-\frac{1}{2}}(3x^{-4})\right]\right]\right\}(3x^2+27)^{1/3}(x^7+1)^8 - \left[8x^5-x^6(2-x^{-3})^{1/2}\right]\left[\frac{1}{3}(3x^2+27)^{-2/3}(6x)(x^7+1)^8+(3x^2+27)^{1/3}\left[8(x^7+1)^7(7x^6)\right]\right]}{\left[(3x^2+27)^{1/3}(x^7+1)^8\right]^2}\right]$ (6)

(corrections in red: x^6, $6x^5$, 6, 6)

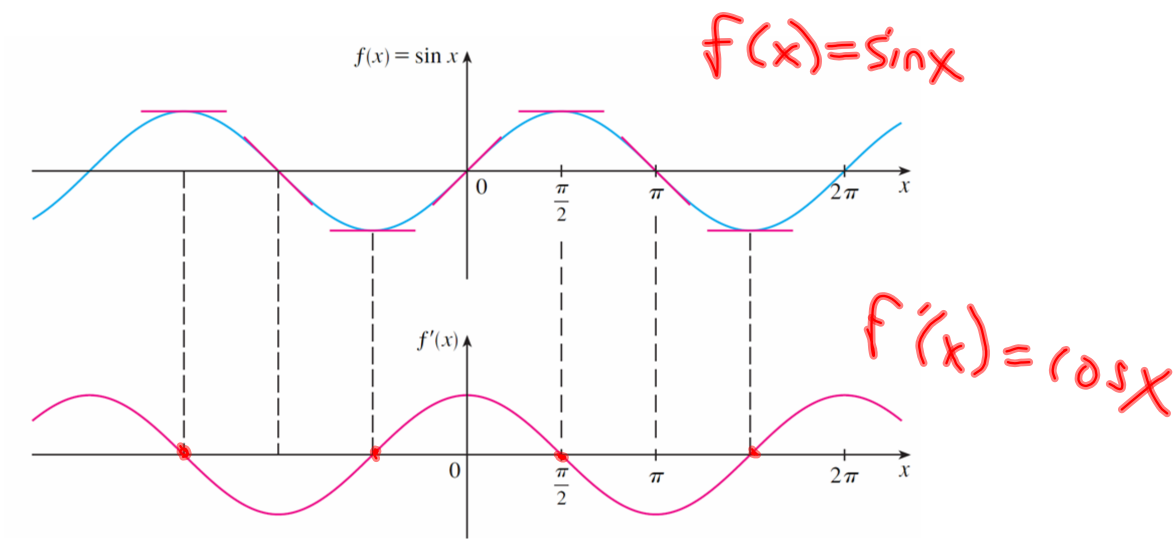
b) $f'(x) = 9\left[8x^3-(x^{10}+(8-5x)(9x^3+5x^{-3})^{1/2})^{1/2}\right]^8\left[24x^2-\frac{1}{2}\left[x^{10}+(8-5x)(9x^3+5x^{-3})^{1/2}\right]^{-\frac{1}{2}}\left[10x^9+\right.\right.$

$\left.\left.(-5)(9x^3+5x^{-3})^{1/2}+(8-5x)\left[\frac{1}{2}(9x^3+5x^{-3})^{-\frac{1}{2}}(27x^2-15x^{-4})\right]\right]\right]$ (6)

Derivatives of Trigonometric Functions

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$f(x) = \sin x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h \to 0} \sin x = \sin x \text{ and } \lim_{h \to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h \to 0} \frac{\sin h}{h} = 1 \text{ and } \lim_{h \to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$\frac{d}{du}(\sin u) = \cos u \bullet du$	$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$
$\frac{d}{du}(\cos u) = -\sin u \bullet du$	$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$
$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$	$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$

eg. $y = \tan(x^4)$

$y' = \sec^2(x^4)(4x^3)$

$y = \csc\sqrt{x+3}$

$y' = -\csc\sqrt{x+3}\cot\sqrt{x+3}\left(\frac{1}{2}(x+3)^{-\frac{1}{2}}\right)$

Differentiate each of the following:

1. $f(x) = \cos\sqrt{5x-1} + \tan x^3$

2. $y = \dfrac{\sec(5x)}{\cot\sqrt{x}}$

3. $f(x) = \csc^2\sqrt{x} - \sqrt{\sin(9x^6)}$

4. $f(x) = \tan\left[\cos\left(8x^{-3}\right)\right]$

5. $f(x) = \sin\left\{\cos\left[\tan^3(7x)\right]\right\}$

6. $y = \dfrac{6x^3\sqrt{5\cot\sqrt{x} + \cos^3 3x}}{\tan\left(\sin\sqrt[3]{x}\right) + 8\cot x^7 - \csc\left(x^4 - 1\right)^5}$

Homework

Worksheet on derivatives of trigonometric functions

Attachments

Derivatives Worksheet.doc