Warm Up

The Mean Value Theorem guarantees the existence of a special point on the graph of $y=\sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of this point?

(A) $(2,1)$

(B) $(1,1)$

(C) $(2,\sqrt{2})$

(D) $\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$

(E) None of the above

$m=\frac{2}{4}=\frac{1}{2} \leftarrow$ slope of secant

$y'=\frac{1}{2}x^{-1/2} \leftarrow$ slope of tangent

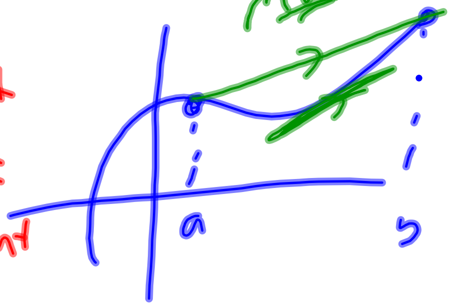
$\frac{1}{2\sqrt{x}}=\frac{1}{2}$

$2\sqrt{x}=2$

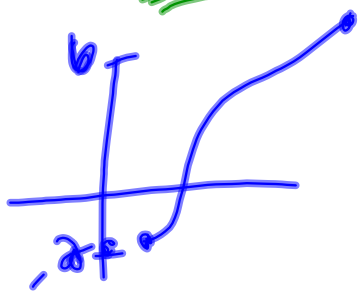
$\sqrt{x}=1$

$x=1$

MVT

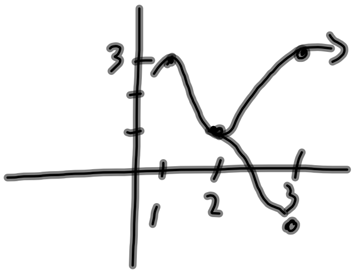


IVT



x	1	2	3
y	3	1	c

$\cdot -1$

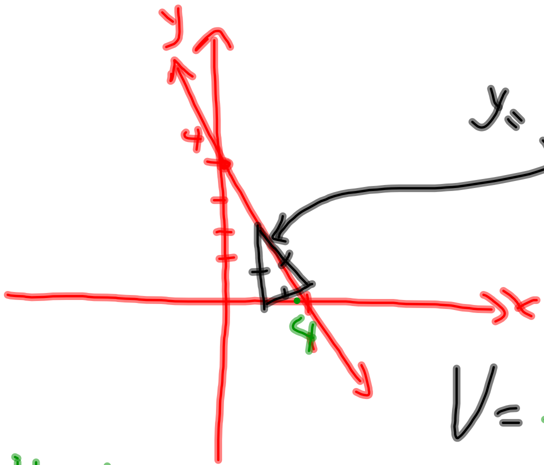


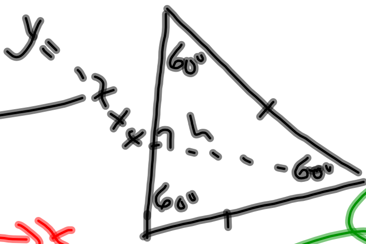
What value of "c" by IVT would guarantee at least one solution to the equation $f(x)=0$.

A/ 0 B/ -1 C/ 3 D/ 2

Example 15 A solid has its base is the region bounded by the lines x + y = 4, x = 0 and y = 0 and the cross section is perpendicular to the x-axis are equilateral triangles. Find its volume.

$y=-x+4$





$A=\frac{1}{2}bh$

OR

$A=\frac{1}{2}ab\sin\theta$

$y=-x+4$

$0=-x+4$

$-4=-x$

$x=4$

$$V=\frac{1}{2}\int_0^4(-x+4)^2\sin 60^\circ\,dx$$

$$=\frac{\sqrt{3}}{4}\int_0^4(x^2-8x+16)\,dx$$

$$=\frac{\sqrt{3}}{4}\left(\frac{x^3}{3}-4x^2+16x\right)\Big|_0^4$$

$$=\frac{\sqrt{3}}{4}\left(\frac{64}{3}-\cancel{64}+\cancel{64}\right)-0$$

$$=\frac{\sqrt{3}}{\cancel{4}}\left(\frac{\cancel{64}^{\,16}}{3}\right)$$

$$=\frac{16\sqrt{3}}{3}$$

Warm Up...

7. Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$ whose cross sections perpendicular to the x-axis are ~~equilateral triangle~~s.

semi-circles

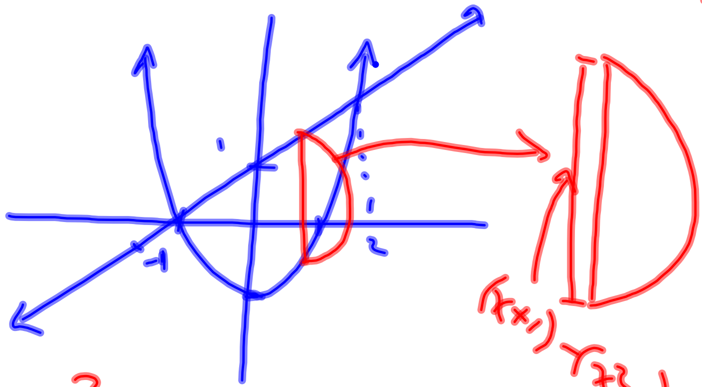
$x + 1 = x^2 - 1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

$V = \frac{1}{2}\pi r^2$



$\underbrace{(x+1) - (x^2 - 1) = -x^2 + x + 2}_{\text{diameter}}$

$r = \frac{-x^2 + x + 2}{2}$

$$V = \frac{\pi}{2}\int_{-1}^{2}\left(\frac{-x^2 + x + 2}{2}\right)^2 dx$$

$$V = \frac{\pi}{8}\int_{-1}^{2}(-x^2 + x + 2)^2 dx$$

$(-x^2 + x + 2)(-x^2 + x + 2)$

$x^4 - x^3 - 2x^2 - x^3 + x^2 + 2x$
$-2x^2 + 2x + 4$

$$= \frac{\pi}{8}\int_{-1}^{2}(x^4 - 2x^3 - 3x^2 + 4x + 4)\, dx$$

Midterm Review:

- Antiderivatives → Application
- L'Hospital's Rule
- Intermediate Value Theorem
- Mean Value Theorem
- Area and Volume

  Approximations using rectangles
  Approximation using trapezoidal rule
  Riemann Summation
  Area bound by curves
  Volumes of revolution...
    -disks
    -washers
    -shell method
  Volumes from known cross-sections

Review Package:

50 Multiple Choice Questions from AP Exams

Attachments

Review of antiderivatives, area and volume.doc