Midterm I

Avg. $\Rightarrow$ 78%

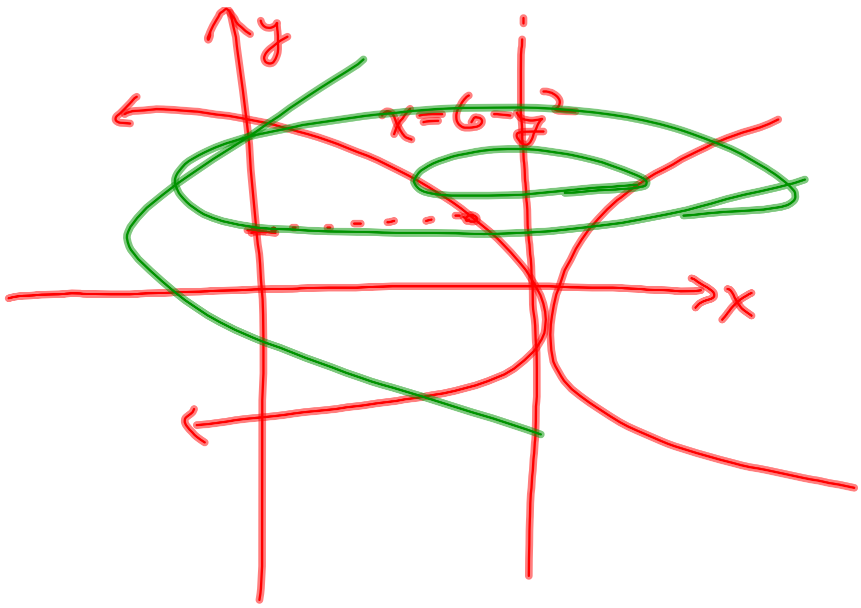
Report Card

Midterm $\Rightarrow$ 80%

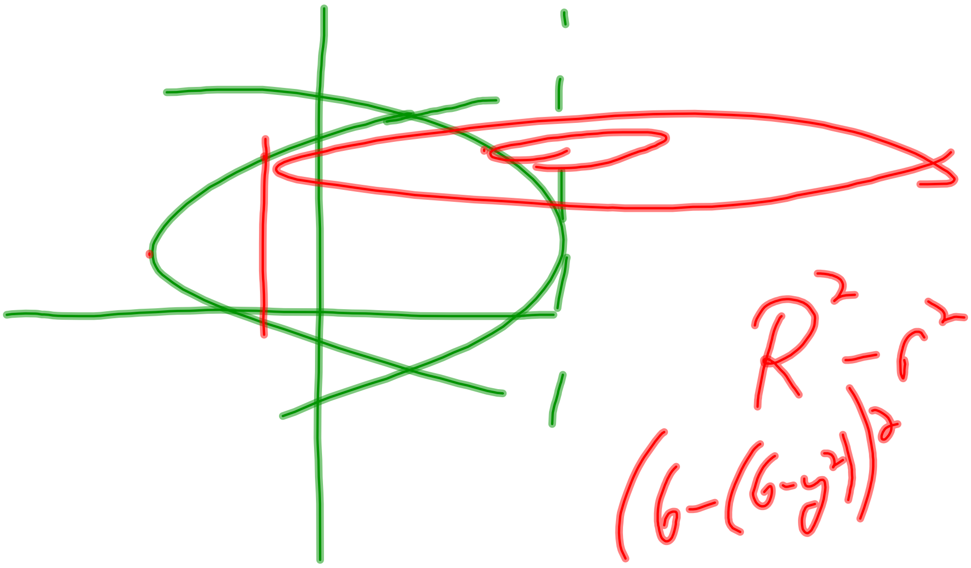
Quiz $\Rightarrow$ 20%

Final Evaluation

$$\left.\begin{array}{l} \text{I} \rightarrow 25\% \\ \text{II} \rightarrow 25\% \\ \text{Qu, 2, Ass}: 20\% \end{array}\right\} 70\%$$

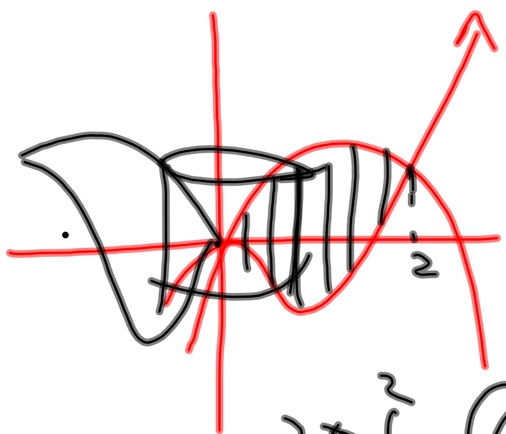


y
x
x = 6 − y²



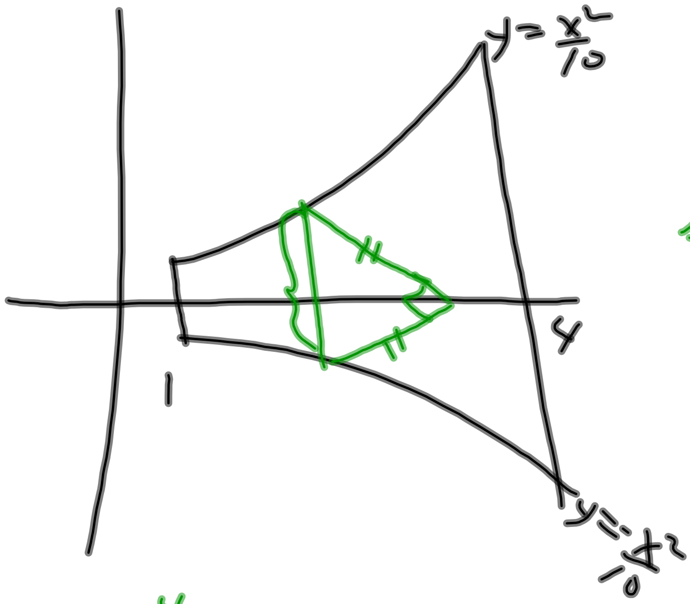
$$R^2 - r^2$$

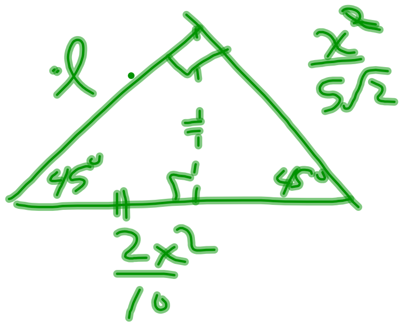
$$(6-(6-y^2))^2$$



$$2\pi\int_0^2 x\left[(4x-x^2)-(x^3-2x)\right]dx$$

$$\frac{56\pi}{5}$$





$$l^2+l^2=\left(\frac{2x^2}{10}\right)^2$$

$$2l^2=\frac{4x^4}{100}$$

$$l^2=\frac{4x^4}{200}$$

$$l=\frac{\cancel{2}x^2}{\cancel{10}_{5}\sqrt{2}}$$

$$l=\frac{x^2}{5\sqrt{2}}$$

$$V=\frac{1}{2}\int_1^4\left(\frac{x^2}{5\sqrt{2}}\right)^2dx$$

$$V=\frac{1}{2}\int_1^4\frac{x^4}{50}$$

$$V=\frac{1}{100}\int_1^4 x^4$$

$$=\frac{256}{125}u^3$$

Applying Integrals to Rate of Change

Example 1:

If V(t) is the volume of water in a reservoir at any time t, then its derivative is the rate at which water flows into the reservoir at time t.

What would the following integral represent?

$$\int_{t_1}^{t_2} V'(t)\,dt = V(t_2) - V(t_1)$$

This integral would represent the change in the amount of water in the reservoir between t_1 and t_2.

Example 2:

If an object moves along a straight line with a position function s(t), then its velocity is v(t) = s$'$(t).

What would the following integral represent?

$$\int_{t_1}^{t_2} v(t)\,dt = s(t_2) - s(t_1)$$

This would represent the net change in displacement, of the particle during the time period from t_1 to t_2.

- If we want to calculate the distance traveled during the time interval, we have to consider the intervals both when
  - $v(t) \geq 0$ (the particle moves to the right) and
  - $v(t) \leq 0$ (the particle moves to the left).
- In both cases the distance is computed by integrating $|v(t)|$, the speed. Therefore

$$\int_{t_1}^{t_2} |v(t)|\,dt = \text{ total distance traveled}$$

Example:

A particle moves along a line so that its velocity at any time t is given by $v(t) = t^2 - t - 6$ m/s.

(a) Determine the displacement of the particle over the interval $1 \leq t \leq 4$ seconds.

(b) Determine the distance traveled over the same time interval.

Look at a diagram mapping the particle's path as well

$v = t^2 - t - 6$

$v = (t-3)(t+2)$

$t = 3, -2$

a) $\int_1^4 (t^2 - t - 6)\,dt$

$= \left. \frac{t^3}{3} - \frac{t^2}{2} - 6t \right|_1^4$

$= \left(\frac{64}{3} - 8 - 24\right) - \left(\frac{1}{3} - \frac{1}{2} - 6\right)$

$= \left(\frac{64}{3} - 32\right) - \left(\frac{1}{3} - \frac{1}{2} - 6\right)$

$= 21 - 32 + \frac{1}{2} + 6$

$= -4.5\text{ m}$

b) $\int_1^3 |t^2 - t - 6|\,dt + \int_3^4 |t^2 - t - 6|\,dt$

$= \left| \left(\frac{t^3}{3} - \frac{t^2}{2} - 6t\right)\Big|_1^3 \right| + \left| \left(\frac{t^3}{3} - \frac{t^2}{2} - 6t\right)\Big|_3^4 \right|$

$= \left| \left(9 - \frac{9}{2} - 18\right) - \left(\frac{1}{3} - \frac{1}{2} - 6\right) \right| + \left| \left(\frac{64}{3} - 8 - 24\right) - \left(9 - \frac{9}{2} - 18\right) \right|$

$= \frac{61}{6}\text{ m}$

Fundamental Theorem of Calculus

- The first part of the FTC deals with functions of the form

$$g(x) = \int_a^x f(t)\,dt$$

where f is a continuous function on $[a, b]$ and x varies between a and b.

- If f happens to be a <u>positive</u> function, then what would $g(x)$ represent??

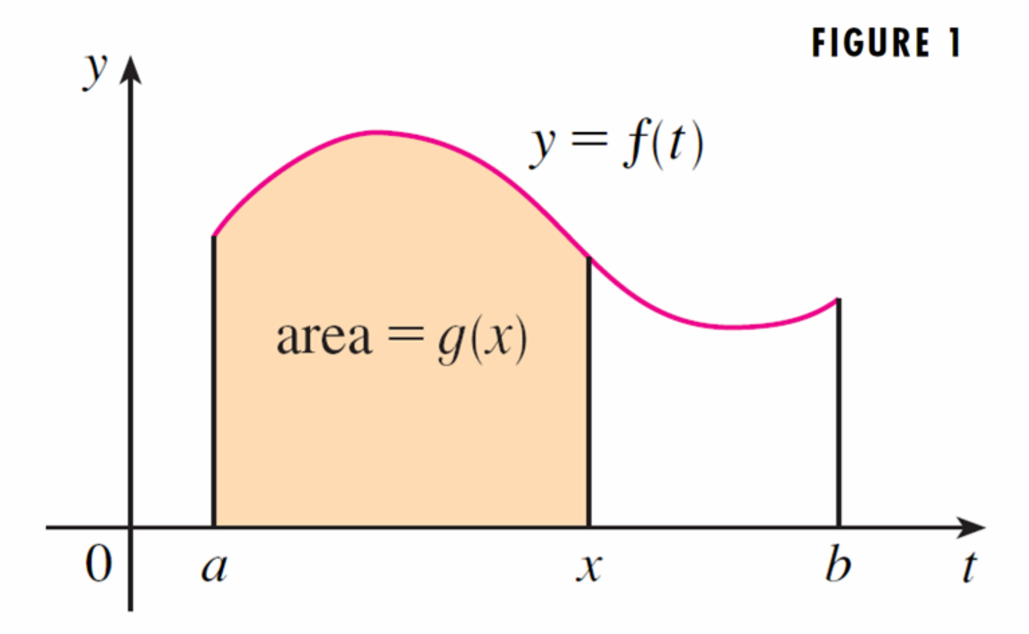


FIGURE 1

If f is the function shown below and $g(x)=\int_a^x f(t)dt$, find the values of $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(4)$ and $g(5)$.

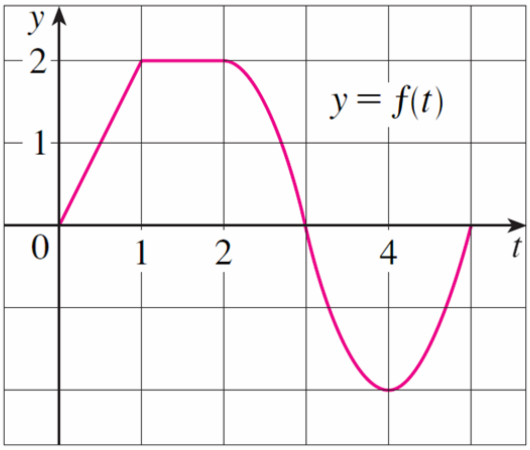


FIGURE 2

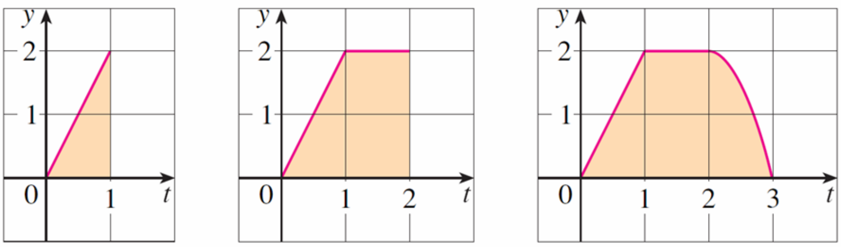
Let's look at g(0) and g(1)...

$$g(2) = \int_0^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt = 1 + (1 \cdot 2) = 3$$

$$g(3) = g(2) + \int_2^3 f(t)\,dt \approx 3 + 1.3 = 4.3$$

$$g(4) = g(3) + \int_3^4 f(t)\,dt \approx 4.3 + (-1.3) = 3.0$$

$$g(5) = g(4) + \int_4^5 f(t)\,dt \approx 3 + (-1.3) = 1.7$$



$g(1) = 1$ $\quad$ $g(2) = 3$ $\quad$ $g(3) \approx 4.3$

The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\,dt \qquad a \leqslant x \leqslant b$$

is an antiderivative of f, that is, $g'(x) = f(x)$ for $a < x < b$.

In Leibniz notation...

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

This is saying that integration and differentiation are inverses of one another.

Evaluate the following:

$$\frac{d}{dx}\int_1^x t^3\,dt$$

According to above, should equal?? x^3

Let's check using our traditional methods...

- Integrate and then differentiate

More Examples:

$$\frac{d}{dx}\int_{-\pi}^{x} \cos t\,dt =$$

$$\frac{d}{dx}\int_0^x \frac{1}{1+t^2}\,dt =$$

upper bound is not *x*...now what??

$$\frac{d}{dx}\int_1^{x^2} \cos t\,dt$$

Let $u = x^2$ and apply the chain rule when finding $\frac{dy}{dx}$ ...

$$\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$$

$$\frac{dy}{dx} = \left(\frac{d}{du}\int_1^{u} \cos t\,dt\right) \bullet \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \bullet \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos x^2 \bullet 2x$$

Now let's try and do these a little quicker...

$$\frac{d}{dx}\int_0^{x^2} e^{t^2}\,dt =$$

$$\frac{d}{dx}\int_0^{5x} \frac{\sqrt{1+t^2}}{t}\,dt =$$

Here are a couple with a little twist...

$$\frac{d}{dx}\int_{x}^{5} 3t\sin t\,dt =$$

Lower bound is not a constant???

Compare these...

$$\int_{1}^{3} x^2\,dx =$$

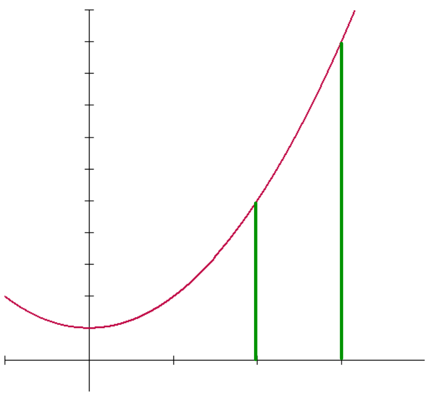
$$\int_{3}^{1} x^2\,dx =$$

$$\frac{d}{dx}\int_{2x}^{x^2} \frac{1}{2+e^t}\,dt =$$

Neither bound is a constant???

Use this type of reasoning...

$$\int_{1}^{3} (x^2+1)\,dx = \int_{0}^{3}\left(x^2+1\right)dx - \int_{0}^{1}\left(x^2+1\right)dx$$



Example:

Find $g(1)$, given that $g(x)=\frac{d}{dx}\int_{x^2}^{x^3}\left(3t-t^3\right)dt$

Practice problems...

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#3, 7, 9, 11, 13, 15, 17, 21

Attachments

Worksheet - Nature of the Roots.doc