Warm Up

Evaluate each of the following:

$$\int e^{2t}\sqrt{1+e^{2t}}\,dt$$

$$\int_1^e x^2 \ln x \; dx$$

This one requires a clever strategy...see if you can figure it out??

$$\int e^x \cos x dx$$

$$\int_0^{1/2} \arcsin x dx = \quad ?.$$

Practice...

Page 498

#3 - 24 (odd numbers)

Bonus

$$\int \frac{12t^2 + 36}{\sqrt[5]{3t + 2}}\, dt$$

Answer....NOT A CHANCE I move this box today!!!

Review of Integration...

Evaluate each of the following:

$$\int \frac{x+4}{x^2+4}\,dx$$

$\frac{1}{2}\int \frac{2x}{x^2+4}dx + \int \frac{4}{x^2+4}dx$

$\frac{1}{2}\ln(x^2+4) + 4\int \frac{dx}{4(\frac{1}{4}x^2+1)}$

$+ 2\int \frac{(\frac{1}{2})dx}{(\frac{1}{2}x)^2+1}$

$= \frac{1}{2}\ln(x^2+4) + 2\tan^{-1}(\frac{1}{2}x) + C$

$$\int x^2\cos(5x)\,dx$$

x^2 (+)	$\cos(5x)$
$2x$ (−)	$\frac{1}{5}\sin(5x)$
2 (+)	$-\frac{1}{25}\cos 5x$
0	$-\frac{1}{125}\sin 5x$

$= \frac{1}{5}x^2\sin(5x) + \frac{2}{25}x\cos 5x - \frac{2}{125}\sin 5x + C$

$$\int \tan^3 x\sec^4 x\,dx$$

$\int \tan^3 x\sec^2 x\sec^2 x\,dx$

$\int \tan^3 x(1+\tan^2 x)\sec^2 x\,dx$

$\int (\tan^3 x + \tan^5 x)\sec^2 x\,dx$

$\int (\tan^3 x\sec^2 x + \tan^5 x\sec^2 x)dx$

$u^n \cdot du \qquad u^n \cdot du$ ☺

$= \frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C$

$$\int_1^e \ln x\,dx$$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$= x\ln x - \int x(\frac{1}{x}dx)$

$= x\ln x - \int dx$

$= x\ln x - x\Big|_1^e$

$= (e\ln e - e) - (1\ln 1 - 1)$

$= e - e + 1$

$= 1$

Integrals Involving Trigonometry

$$\int \sin x \cos^3 x \, dx =$$

$$-\int \cos^3 x (\sin x \, dx)$$

$$= -\frac{1}{4}\cos^4 x + C$$

$$\int \sin x \cos^2 x \cos x \, dx$$

$$\int \sin x (1 - \sin^2 x) \cos x \, dx$$

$$\int (\sin x - \sin^3 x) \cos x \, dx$$

$$\int \sin x \cos x \, dx - \int \sin^3 x \cos x \, dx$$

$$= \frac{1}{2}\sin^2 x - \frac{1}{4}\sin^4 x + C$$

These identities look familiar??

$$\sin^2 x = \frac{1-\cos 2x}{2}, \quad \cos^2 x = \frac{1+\cos 2x}{2}, \quad \sin x \cos x = \frac{1}{2}\sin 2x$$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin 2x = 2\sin x\cos x$

1 $\cos 2x = 1 - 2\sin^2 x$

$\frac{1}{2}\sin 2x = \sin x\cos x$

$\cos 2x = 2\cos^2 x - 1$

They might help with one like this...

$\int \sin^2 x \, dx$ ← Substitute a Double Angle Identity

$\int \frac{1-\cos 2x}{2} dx$

$\int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx$

$\int \frac{1}{2} dx - \frac{1}{2}\cdot\frac{1}{2}\int \cos 2x (2) dx$

$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

Integration using Trigonometric Substitution

- Method of integration used to evaluate integrals involving...

$$\sqrt{x^2 - a^2} \qquad \sqrt{x^2 + a^2} \qquad \sqrt{a^2 - x^2}$$

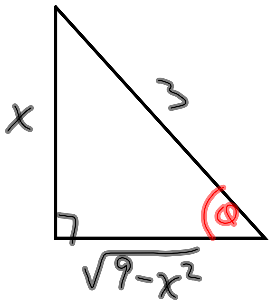
Example:

Represent $\sqrt{9 - x^2}$ using a trigonometric ratio.

$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

$3\cos\theta = \sqrt{9-x^2}$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$



- Express θ as an inverse trigonometric ratio

Example: $\int \frac{dx}{\sqrt{9 - x^2}}$

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Inspection

$\int \frac{dx}{\sqrt{9(1-\frac{1}{9}x^2)}}$

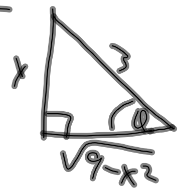
$\frac{1}{3}\int \frac{\left(\frac{1}{3}\right)dx}{\sqrt{1-\left(\frac{1}{3}x\right)^2}}$

$= \sin^{-1}\left(\frac{1}{3}x\right) + C$

Trig. Substitution

$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

$3\cos\theta = \sqrt{9-x^2}$



$\sin\theta = \frac{x}{3}$

$3\sin\theta = x$

$3\cos\theta = \frac{dx}{d\theta}$

$3\cos\theta\, d\theta = dx$

$\int \frac{3\cos\theta\, d\theta}{3\cos\theta}$

$\int d\theta$

$= \theta$

$\theta = \sin^{-1}\frac{x}{3}$

$= \sin^{-1}\frac{x}{3} + C$

OR

$= \tan^{-1}\frac{x}{\sqrt{9-x^2}} + C$

Example:

$$\int \frac{dx}{x\sqrt{x^2 - 16}}$$

[redacted]

Attachments

Worksheet - Intro. to Average Rate of Change.doc

Derivatives Worksheet.doc