Warm Up

Evaluate each of the following:

$$\int \frac{x+4}{x^2+4}\,dx$$

$$\int x^2\cos(5x)\,dx$$

$$\int \tan^3 x\sec^4 x\,dx$$

$$\int_1^e \ln x\,dx$$

Integrals Involving Trigonometry

$$\int \sin x \cos^3 x \, dx =$$

These identities look familiar??

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}, \quad \sin x \cos x = \frac{1}{2}\sin 2x$$

They might help with one like this...

$$\int \sin x \cos^3 x \, dx =$$ [redacted]

Bonus

$$\int \frac{12t^2 + 36}{\sqrt[5]{3t + 2}}\,dt$$

$u = 12t^2+36$	$+$	$dv = \frac{1}{3}(3t+2)^{-\frac{1}{5}}(3)$
$24t$	$-$	$\frac{5}{12}(3t+2)^{4/5}$
24	$+$	$\frac{25}{324}(3t+2)^{9/5}$
0		$\frac{125}{13608}(3t+2)^{14/5}$

$$= \frac{5}{12}(12t^2+36)(3t+2)^{4/5} - \frac{25}{324}(24t)(3t+2)^{9/5} + 24\left(\frac{125}{13608}\right)(3t+2)^{14/5} + C$$

Answer: $(5t^2 + 15)(3t + 2)^{4/5} - \frac{50t}{27}(3t + 2)^{9/5} + \frac{125}{567}(3t + 2)^{14/5} + C$

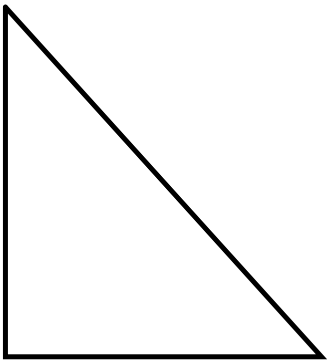
Integration using Trigonometric Substitution

- Method of integration used to evaluate integrals involving...

$\sqrt{x^2 - a^2}$ $\sqrt{x^2 + a^2}$ $\sqrt{a^2 - x^2}$

Example:

Represent $\sqrt{9 - x^2}$ using a trigonometric ratio.



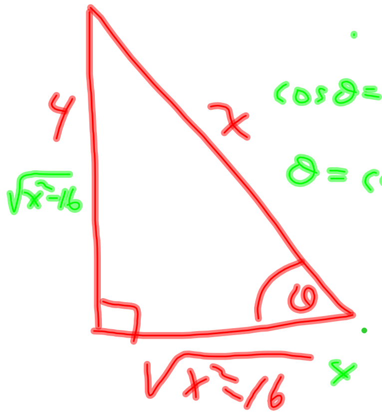
- Express θ as an inverse trigonometric ratio

Example: $\int \frac{dx}{\sqrt{9 - x^2}}$

Example:

$$\int \frac{dx}{x\sqrt{x^2-16}}$$

[redacted]



$\cos\theta = \frac{4}{x}$

$\theta = \cos^{-1}\left(\frac{4}{x}\right)$

$\tan\theta = \frac{4}{\sqrt{x^2-16}}$

$\sqrt{x^2-16} = \frac{4}{\tan\theta}$

$= 4\cot\theta$

$\sin\theta = \frac{4}{x}$

$x = \frac{4}{\sin\theta}$

$x = 4\csc\theta$

$dx = -4\csc\theta\cot\theta\, d\theta$

$\frac{1}{4}x = \csc\theta$

$\therefore \csc^{-1}\left(\frac{1}{4}x\right) = \theta$

$$\int \frac{-\cancel{4}\cancel{\csc\theta}\cancel{\cot\theta}\,d\theta}{(\cancel{4}\cancel{\csc\theta})(4\cancel{\cot\theta})}$$

$-\frac{1}{4}\int d\theta$

$= -\frac{1}{4}\theta + C$

$= -\frac{1}{4}\csc^{-1}\left(\frac{1}{4}x\right) + C$

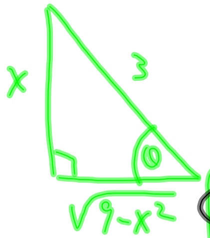
$y = \cos^{-1}\left(\frac{4}{x}\right)$ $\quad 4x^{-1}$

$$y' = \frac{-4x^{-2}}{\sqrt{1-\left(\frac{16}{x^2}\right)}}$$

Example:

$$\int \frac{x^2\,dx}{\sqrt{9-x^2}}$$

Hint: Will require trigonometric identities



$\sin\theta=\frac{x}{3}$

$3\sin\theta=x$

$\boxed{9\sin^2\theta=x^2}$

$\boxed{3\cos\theta\,d\theta=dx}$

$\cos\theta=\frac{\sqrt{9-x^2}}{3}$

$\boxed{3\cos\theta=\sqrt{9-x^2}}$

$$\int\frac{9\sin^2\theta\,(\cancel{3}\cancel{\cos\theta})\,d\theta}{\cancel{3}\cancel{\cos\theta}}$$

$\cos 2\theta=2\cos^2\theta-1$

$\cos 2\theta=1-2\sin^2\theta$

$\sin^2\theta=\frac{1-\cos 2\theta}{2}$

$$9\int\sin^2\theta\,d\theta$$

$$\frac{9}{2}\int(1-\cos 2\theta)\,d\theta \longrightarrow \int(1-\cos^2\theta)\,d\theta$$

$$\frac{9}{2}\int d\theta-\frac{9}{2\cdot 2}\int\cos 2\theta\overset{(2)}{d\theta}\quad=\theta-$$

$$=\frac{9}{2}\theta-\frac{9}{4}\underline{\sin 2\theta}+C$$

$$=\frac{9}{2}\theta-\frac{9}{\cancel{4}_{2}}(\cancel{2}\sin\theta\cos\theta)+C$$

$$=\frac{9}{2}\sin^{-1}\frac{x}{3}-\frac{\cancel{9}}{2}\left(\frac{x}{\cancel{3}}\right)\left(\frac{\sqrt{9-x^2}}{\cancel{3}}\right)+C$$

$$=\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)-\frac{1}{2}x\sqrt{9-x^2}+C$$

OR

$$=\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)-\frac{9}{4}\sin\left(2\left(\sin^{-1}\frac{x}{3}\right)\right)+C$$

$$\int \frac{x^2}{x^2+1}\,dx$$

$$\Rightarrow \begin{array}{r} 1 \\ x^2+1 \overline{)\, x^2 \quad} \\ \underline{x^2+1} \\ -1 \end{array}$$

$$\int\left(1-\frac{1}{x^2+1}\right)dx$$

$$\int 1\,dx - \int\frac{dx}{x^2+1}$$

$$= x - \tan^{-1}x + C$$

$$\frac{7}{6} = \begin{array}{r} 1 \\ 6\overline{)\,7} \\ \underline{6} \\ ① \end{array}$$

$$\frac{7}{6} = 1+\frac{1}{6}$$

$$1\frac{1}{6}$$

$$\int \frac{x^2+1}{x^2}\,dx$$

$$\int \frac{x^2}{x^2}\cancel{dx} \quad \int \frac{1}{x^2}\,dx$$

$$\int \frac{dx}{x^2-4x+5}$$

completing square !!

$$\int \frac{dx}{\left(x^2-4x+\underline{4}\right)+5-\cancel{4}}$$

$$\int \frac{dx}{(\underbrace{x-2})^2+1} \qquad \frac{du}{u^2+1}$$

$$= \tan^{-1}(x-2)+C$$

Integration using Partial Fractions

Simplify: $\dfrac{3}{x-5}+\dfrac{2}{x+4}$

We want to reverse the process of finding a common denominator...

Express as partial fractions: $\dfrac{5x+2}{x^2-x-20}$

1. Factor the denominator: $\dfrac{5x+2}{(x-5)(x+4)}$

2. Separate into partial fractions:

$$\frac{A}{x-5}+\frac{B}{x+4}=\frac{5x+2}{x^2-x-20}$$

3. Find common denominator and solve for A and B:

Now let's evaluate the following integral...

$$\int \frac{(5x+2)dx}{x^2 - x - 20}$$

Here is another example...

$$\int \frac{xdx}{x^2 - 3x + 2}$$

Practice Questions...

Warm Up

Evaluate each of the following:

$$\int \frac{1}{(16 - x^2)^{\frac{3}{2}}}\, dx$$ (UNB: 2005)

$$\int (\csc^2 t \cot^2 t)\, dt$$ (UNB: 2004)

$$\int \frac{\ln(\ln x)}{x}\, dx$$ (StFX: 2005)