Quiz: Quadratics 36 *Functions and Relations 112* *Name:* Solutions

March 2012

I. **Multiple Choice**: ***Place the letter corresponding to the correct solution on the space provided.***

B 1. The vertex of the parabola $y = -3x^2 + 18x + 10$ would be located at the ordered pair…

[A] (-3 , -37) [B] (3 , 37) [C] (-3 , 1) [D] (3 , -1)

$y=-3(x^2-6x+9)+10+27$

$(x-3)^2+37$

A 2. If the graph of $y = -5(x+3)^2 + 6$ is drawn, which of the following is **not** a possible value of y on the graph?

[A] 8 [B] 1 [C] –2 [D] –10

$v(-3,6)$

B 3. Which of the following is a root of the equation $-x^2 + 2x = -2$?

[A] $\frac{-2-\sqrt{12}}{2}$ [B] $1+\sqrt{3}$ [C] $1+\sqrt{12}$ [D] none of these

$x^2-2x-2=0$

$x=\frac{2\pm\sqrt{4-4(1)(-2)}}{2}$

$x=\frac{2\pm\sqrt{12}}{2}$

$\frac{2\pm2\sqrt{3}}{2}$

$1\pm\sqrt{3}$

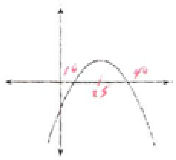
A 4. The height above the ground of a swooping hawk is given by $h(t) = 3t^2 - 30t + 100$ where h is the height in meters above the ground and t is the time in seconds. How high above the ground is the hawk after 10 seconds?

[A] 100 m [B] 25 m [C] 9 m [D] 75 m

$=3(100)-300+100$

C 5. The x-intercepts of the parabola shown are (10, 0) and (40, 0)

Which of the following could be the vertex?



[A] (30 , 10) [B] (25, -8)

[C] (25, 10) [D] (30, 10)

B 6. The daily revenue of R dollars for a ski resort can be modeled by the equation

$$R = -14t^2 - 280t + 4200$$

where t represents the temperature in degrees Celsius. What is the maximum daily revenue that the resort could earn?

[A] \$4200 [B] \$5600 [C] \$2800 [D] \$9800

$R=-14(t^2-20t+100)+4200+1400$

$(t-10)^2+5600$

$(10, 5600)$

A 7. The roots of the quadratic equation $x^2 - 13x = 30$ are…

[A] 15 and –2 [B] –10 and –3 [C] 10 and 3 [D] –15 and 2

$x^2-13x-30=0$

$(x-15)(x+2)=0$

$x=15,-2$

D 8. While working on an electric generator, a physicist used the quadratic function

$$P = -I^2 + 10I$$

where P is the power output, in watts, and I is the current, in amperes. According to the function, when the power output P is 24 watts, the current is

[A] 0 or 10 amperes [B] –2 or 12 amperes [C] 2 or –12 amperes [D] 6 or 4 amperes

$-I^2+10I=24$

$I^2-10I+24=0$

$(I-6)(I-4)=0$

$I=6,4$

B 9. Completing the square was used by a student to solve the equation $3x^2 + 10x - 8 = 0$.

The steps used were:

$$\text{Step 1: } 3\left(x^2+\frac{10}{3}x+\frac{25}{9}\right)=8+\frac{25}{3}$$

$$\text{Step 2: } 3\left(x-\frac{5}{3}\right)^2=\frac{49}{3}$$

$$\text{Step 3: } \left(x-\frac{5}{3}\right)^2=\frac{49}{9}$$

$$\text{Step 4: } x=\frac{5}{3}\pm\frac{7}{3}$$

An error was made in one of the steps. Identify the step where the first error occurred.

[A] Step I [B] Step II [C] Step III [D] Step IV

A 10. In a recent soccer game, the ball was kicked by the keeper and followed the path given by $h = -5t^2 + 15t$ (where h = height of the ball in metres, t = time in seconds). How long did it take for the ball to hit the ground?

[A] 3 seconds [B] 5 seconds [C] 1.5 seconds [D] 2.25 seconds

$-5t(t-3)=0$

$t=0,3$

II. Open Response: ***Show all work for each of the following in the space provided.***

1. (a) Determine the zeros of the following quadratic function: $h(x) = 4x^2 + 4x - 15$ [4]

$4x^2 + 4x - 15 = 0$

$4x^2 + 10x - 6x - 15 = 0$

$2x(2x+5) - 3(2x+5) = 0$

$(2x+5)(2x-3) = 0$

$2x+5=0 \qquad 2x-3=0$

$2x=-5 \qquad 2x=3$

$\boxed{x=-\frac{5}{2}} \qquad \boxed{x=\frac{3}{2}}$

(b) Solve the following equation: $3(x-1)^2 + 10(x-1) = 8$ [4]

$3(x^2-2x+1) + 10x - 10 - 8 = 0$

$3x^2 - 6x + 3 + 10x - 18 = 0$

$3x^2 + 4x - 15 = 0$

$3x^2 + 9x - 5x - 15 = 0$

$3x(x+3) - 5(x+3) = 0$

$(x+3)(3x-5) = 0$

$\boxed{x=-3} \qquad \boxed{x=\frac{5}{3}}$

(c) Determine the roots of the following quadratic equation: $\frac{3}{3-x} - \frac{4}{x+2} = -2$ $(3-x)(x+2)$ [4]

$3(x+2) - 4(3-x) = -2(3x+6-x^2-2x)$

$3x+6-12+4x = -6x-12+2x^2+4x$

$7x-6 = 2x^2-2x-12$

$2x^2 - 9x - 6 = 0$

$x = \frac{9 \pm \sqrt{81-4(2)(-6)}}{2(2)}$

$\boxed{x = \frac{9 \pm \sqrt{129}}{4}}$

$x = 5.089$ or $x = -0.589$

3. A student in Phys. Ed. class leans forward and plunges into the pool head first from the diving board and then resurfaces. The student follows a parabolic path defined by the function $d(t) = t^2 - 4t + 1$, where t is the time, in seconds, that has elapsed since leaving the diving board, and d is the height above or below the water in metres.
(Remember that below the water will be represented by a negative depth)

(a) How high is the diving board above the water? [1]

$d(0) = 0 - 0 + 1 = \boxed{1\ m}$

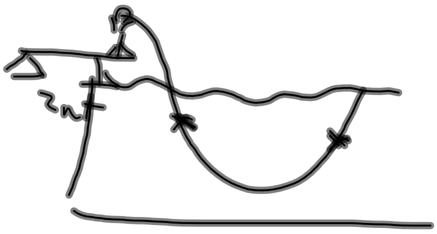
(b) How long will it take the student to reach a depth of 2 m below the surface? [4]

$t^2 - 4t + 1 = -2$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3$ or 1



1 second

(c) How long after leaving the diving board will the student ~~resurface~~ from below the water? [3]

$t^2 - 4t + 1 = 0$

$t = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2}$

$t = \frac{4 \pm \sqrt{12}}{2}$

$t = \frac{4 \pm 2\sqrt{3}}{2}$

$t = 2 \pm \sqrt{3}$

$t = 3.73$ sec and 0.3

(d) What is the minimum depth below the surface that the student will reach throughout the course of the dive? [3]

$d(t) = t^2 - 4t + 1$

$d(t) = (t^2 - 4t + 4) + 1 - 4$

$y = (t-2)^2 - 3$

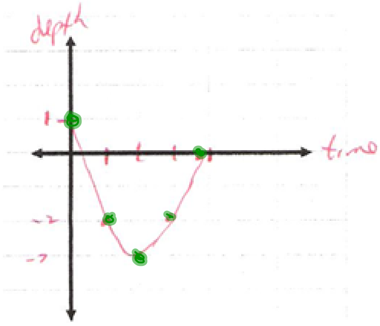
$(2, -3)$

(t, d)

x

3m below

(d) On the axes provided, sketch the path of the student from the time they leave the diving board, until the time they resurface. Clearly label and scale your axes, identify the coordinates of the points used to develop your sketch. [3]



Warm Up

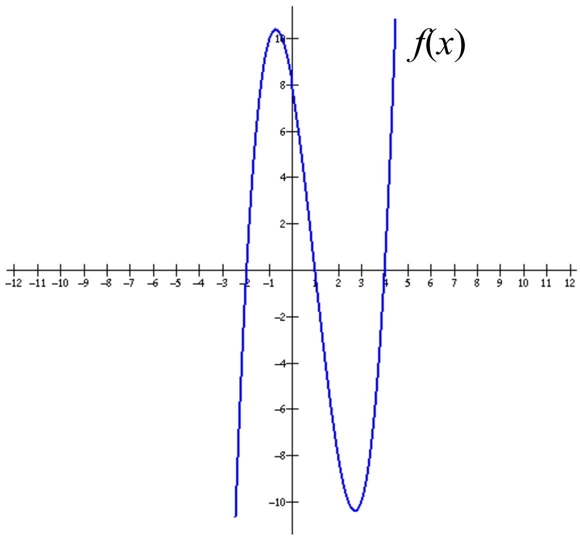
A fashion magazine prints its photographs in two square sizes. The larger photographs measure 3 cm longer than the smaller photographs. Three of the smaller photographs have a combined area 9.0 cm^2 more than the area of one of the larger size. Determine the dimensions of each size of photograph.

Quadratic Inequalities

• Quick review of linear inequalities....

Solve the following: $2x - 5 > 6x + 7$

• Determining solutions to an inequality from a graph....



Using the graph of $f(x)$ shown, determine each of the following:

Where is $f(x) > 0$?

Where is $f(x) \leq 0$?

- Let's look at finding solution sets of quadratic inequalities....

$$\text{Solve: } x^2 - 5x < 14$$

We will look at two different approaches...

I. Using cases:

II. Using a sketch:

Example: (Use both methods!!)

Solve: $3x^2 \geq 13x - 10$

The Nature of the Roots

- any quadratic equation can be solved using the quadratic formula...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant
- enables you to determine the "Nature of the Roots" without actually finding the roots.

$D = b^2 - 4ac$

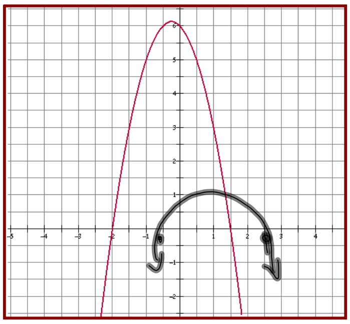
- there are THREE cases...

CASE #1: Real and Unequal Roots (Distinct Roots)

- this happens when Discriminant > 0.
- the quadratic will have two real and unequal roots.

NOTE: If the discriminant is a perfect square, then the roots will be RATIONAL.
Otherwise, the roots will be IRRATIONAL.

EXAMPLE: $2x^2 - x - 6 = 0$



Calculate the discriminant value

$D = b^2 - 4ac$

$D=(-1)^2-4(2)(-6)$

$D= 1+48$

$D=49$

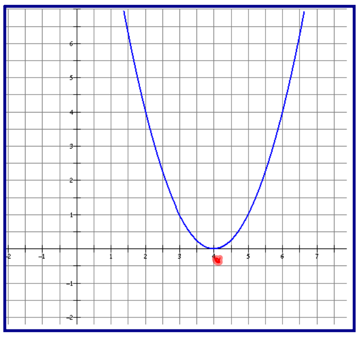
$D>0$ { Perfect Square

Real { Unequal, Rational

CASE #2: Real and Equal Roots

- this happens when Discriminant = 0.
- the quadratic will have two real and equal roots (one real root).

EXAMPLE: $x^2 - 8x + 16 = 0$



Calculate the discriminant value

$D = b^2 - 4ac$

$D=(-8)^2-4(1)(16)$

$=64-64$

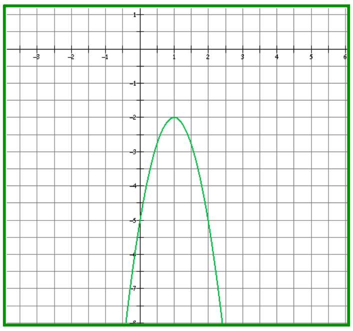
$=0$

Real, Equal, Rational

CASE #3: Non-Real and Unequal Roots

- this happens when Discriminant < 0.
- the quadratic will have two non-real and unequal roots (imaginary/complex roots)

EXAMPLE: $-3x^2 + 6x - 5 = 0$



Calculate the discriminant value

$D = b^2 - 4ac$

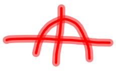
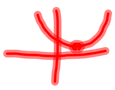
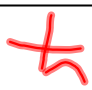
$D=(6)^2-4(-3)(-5)$

$D=36-60$

$D=-24$ } $D<0$

Complex, Unequal

SUMMARY: Nature of the Roots

Value of the Discriminant			
$D = b^2 - 4ac$	**Real or Non-real**	**Equal or Unequal**	**Rational or Irrational**
1. $D > 0$ but not a perfect square 	Real	Unequal	Irrational
2. $D > 0$ and is a perfect square 	Real	Unequal	Rational
3. $D = 0$ 	Real	Equal	Rational
4. $D < 0$ 	Non-real	Unequal	n/a

Sample Problem 1:

Describe the nature of the roots, if

a) $3x^2 + 6x + 1 = 0$

$D = b^2 - 4ac$

$= 6^2 - 4(3)(1)$

$= 36 - 12$

$= 24$

D > 0 but not a perfect square, ∴ roots are real, unequal, and irrational.

b) $x^2 - 5x + 7 = 0$

$D = b^2 - 4ac$

$= (-5)^2 - 4(1)(1)7$

$= 25 - 28$

$= -3$

D < 0, ∴ roots are non-real and unequal.

Another Example: $5x^2 - 5x - 1 = 0$

Sample Problem #2:

Find the value(s) of k so that $x^2 + (k - 1)x - k = 0$
has equal roots.

$D = 0$

$a = 1$
$b = k - 1$
$c = -k$

$x^2 - 2x + 1 = 0$

$(k-1)^2 - 4(1)(-k) = 0$
$k^2 - 2k + 1 + 4k = 0$
$k^2 + 2k + 1 = 0$
$(k+1)(k+1) = 0$
$k = -1$

Sample Problem #3:

Find the value(s) of k so that $x^2 + 12x - k = 0$
has real and unequal roots.

Reverse the sign

HOMEWORK...

Worksheet - Nature of the Roots.doc

Attachments

Worksheet - Nature of the Roots.doc