Avg: 69.3%

Functions and Relations 112

Unit Test: Quadratics
March 2012

Name: Solutions

I. Multiple Choice: Place the letter corresponding to the correct solution on the scantron provided. **[12 Marks]**

1. The coordinates of the vertex of the quadratic function $y = 3x^2 - 6x + 5$ are...

[A] $(1,8)$ [B] $(1,2)$ [C] $(1,6)$ [D] $(1,4)$

$3(x^2 - 2x + 1) + 5 - 3$
$3(x-1)^2 + 2$
$v(1,2)$

2. If a quadratic function has an axis of symmetry of $x = 10$ and a maximum of 8, then the function could be ...

[A] $y = -(x-10)^2 + 8$ [B] $y = (x-10)^2 + 8$ [C] $y = -(x+10)^2 + 8$ [D] $y = -(x-10)^2 - 8$

$v(10, 8)$ opens Down

3. If the graph of $\frac{1}{2}(y+3) = (x-2)^2$ is sketched, what is the range of the graph?

[A] $y \geq 3$ [B] $y \leq -3$ [C] $y \geq -3$ [D] $y \leq 3$

$v(2, -3)$ opens up

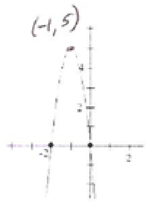
$y + 3 = 2(x-2)^2$
$y = 2(x-2)^2 - 3$
$(2, -3)$

4. For the equation $-5x^2 + x - 2 = 0$, the roots are...

[A] Real and Equal [B] Non-Real [C] Real and Unequal [D] Both Zero

$D = (1)^2 - 4(-5)(-2)$
$D = 1 - 40$
$D = -39$

5. Determine the equation of the quadratic shown:



[A] $y = 5(x+1)^2 + 5$ [B] ~~$y = -5(x+1)^2 - 5$~~

[C] $y = -\frac{1}{5}(x+1)^2 + 5$ [D] $y = -5(x+1)^2 + 5$

6. Which of the following mappings would map the graph of $y = x^2$ to the graph of $y = -5(x-2)^2 + 3$?

[A] $(x, y) \to (x-2, -5y+3)$ [B] $(x, y) \to (x+2, -5y+3)$
[C] $(x, y) \to (-5x+2, y+3)$ [D] $(x, y) \to (5x+2, y-3)$

$v(2,3)$
$a = -5$

7. A baseball is thrown and follows the path given by the relation $h = -4t^2 + 12t$, where h is the height of the Frisbee in metres, and t is the time in seconds. When will the baseball be at a height of 5 m ?

[A] 2.5 seconds [B] 40 seconds [C] 1.5 seconds [D] never

$-4t^2 + 12t = 5$
$4t^2 - 12t + 5$
$4t^2 - 10t - 2t + 5$
$2t(2t-5) - 1(2t-5)$
$(2t-1)(2t-5)$
$t = \frac{1}{2}$ $t = \frac{5}{2}$
2.5 sec

8. Use complex numbers to simplify $3\sqrt{-20}$?

[A] $\pm 5i\sqrt{5}$ [B] $\pm 6i\sqrt{5}$ [C] $\pm 30i\sqrt{2}$ [D] $\pm 12i\sqrt{5}$

$3i\sqrt{20}$ $3i(2)\sqrt{5}$ $6i\sqrt{5}$

9. When the quadratic equation $\frac{1}{2}(y+4) = (x-6)^2$ is written in general form, the equation is

[A] $y = 2x^2 - 24x + 68$ [B] $y = 2(x-6)^2 - 4$ [C] $y = 4x^2 - 48x + 140$ [D] $y = \frac{1}{2}x^2 - 6x + 14$

$y + 4 = 2(x-6)^2$
$y + 4 = 2(x^2 - 12x + 36)$
$y = 2x^2 - 24x + 68$

10. Determine the x-intercepts of the quadratic function $f(x) = x^2 - 13x - 30$

[A] $x = -10$ *and* $x = -3$ [B] $x = -10$ *and* $x = 3$ [C] $x = 15$ *and* $x = -2$ [D] $x = -15$ *and* $x = 2$

$0 = (x-15)(x+2)$
$x = 15, -2$

11. The method of completing the square was used by a student to solve the equation $8x^2 + 10x - 3 = 0$.

The steps used were $8x^2 + 10x - 3 = 0$

Step I: $x^2 + \frac{5}{4}x = \frac{3}{8}$

Step II: $\left(x^2 + \frac{5}{4}x + \frac{25}{64}\right) = \frac{3}{8} + \frac{25}{64}$

Step III: $\sqrt{\left(x + \frac{5}{8}\right)^2} = \sqrt{\frac{49}{64}}$

Step IV: *No Real Solutions*

In which step did this student make their first mistake?

[A] I [B] II [C] III [D] IV

12. The flight path of an eagle can be modeled by a parabola. The eagle reaches a minimum height of 7 metres in a period of 6.25 seconds. Which of the following is a **possible** function for the path relating height, h, to the time, t?

[A] $h = -2(t-6.25)^2 + 7$ [B] $h = 2(t+6.25)^2 + 7$

[C] $h = \frac{1}{2}(t-6.25)^2 + 7$ [D] $h = 2(t-6.25)^2 - 7$

Part II:

Open Response: All work for each of the following is to be shown in the space provided. **[45Marks]**

1. Given the following quadratic function: $y = 5x^2 + 30x + 40$
   Use this function to complete the chart shown below. [13]

$y = 5(x^2 + 6x + 9) + 40 - 45$

$y = 5(x+3)^2 - 5$

Coordinates of Vertex	$(-3,-5)$
y-Intercept	$(0,40)$
Direction parabola opens	up
Equation of axis of symmetry	$x = -3$
Domain	$x \in R$
Range	$y \geq -5$
Mapping Notation	$(x,y) \to (x-3, 5y-5)$
Does function represent a maximum or minimum?	Minimum
What is maximum or minimum value?	$Min = -5$

2. Solve each of the following quadratic equations. ***note: - if roots are non-real, express them as complex roots*** [8]
   - if roots are irrational, simplify the radical completely

(a) $4x^2 - 8x + 5 = 0$

$x = \dfrac{8 \pm \sqrt{64 - 4(4)(5)}}{2(4)}$

$x = \dfrac{8 \pm \sqrt{-16}}{8}$

$i^2 = -1$

$\sqrt{16i^2}$

$x = \dfrac{8 \pm 4i}{8} = \dfrac{8}{8} \pm \dfrac{4}{8}i$

$\boxed{x = 1 \pm \frac{1}{2}i}$

(b) $-\frac{1}{2}(x-4)^2 + 6 = 0$ (-2) (-2) (-2)

$(x-4)^2 - 12 = 0$

$\sqrt{(x-4)^2} = \sqrt{12}$

$x - 4 = \pm 2\sqrt{3}$

$\boxed{x = 4 \pm 2\sqrt{3}}$

$(x-4)^2$

$x^2 - 8x + 16$

or

$-\frac{1}{2}(x^2 - 8x + 16) + 6 = 0$

$-\frac{1}{2}x^2 + 4x - 8 + 6 = 0$

$-\frac{1}{2}x^2 + 4x - 2 = 0$

3. The mass m (in Mg) of fuel supply in the first-stage booster of a rocket is given by the function: $m = -3t^2 + 6t + 140$, where t is the time (in s) after the launch.

(a) What is the mass of fuel in the first-stage booster 3 seconds after liftoff? [1]

$m = -3(3)^2 + 6(3) + 140$

$m = -27 + 18 + 140 = 131$ Mg

(b) According to the function, what is the **maximum** mass of the fuel supply and at what time does this mass occur? [4]

$m = -3(t^2 - 2t + 1) + 140 + 3$

$m = -3(t-1)^2 + 143$

$V(1, 143)$

(t, m)

Maximum Mass = 143 Mg **Time =** 1 Second

(c) At what time will the rocket run out of fuel? ***(Round answer to the nearest tenth of a second)*** [4]

$-3t^2 + 6t + 140 = 0$

$t = \dfrac{-6 \pm \sqrt{36 - 4(-3)(140)}}{2(-3)}$

$t = \dfrac{-6 \pm \sqrt{1716}}{-6}$

$t = \dfrac{-6 \pm 41.4}{-6}$

$t = +7.9$ or $t = -5.9$

Time= 7.9 seconds

(d) At what time(s) will there be 95 Mg of fuel supply remaining in the first-stage booster? [4]

$-3t^2 + 6t + 140 = 95$

$-3t^2 + 6t + 45 = 0$

$t^2 - 2t - 15 = 0$

$(t-5)(t+3) = 0$

$t = 5$ or $^-3$

Time(s) = 5 Seconds

$D > 0$

4. Determine the value(s) of k so that the quadratic equation $(k-3)x^2 - 8x + 2 = 0$ will have **real and unequal** roots. [4]

$a = k - 3$

$b = -8$

$c = 2$

$(-8)^2 - 4(k-3)(2) > 0$

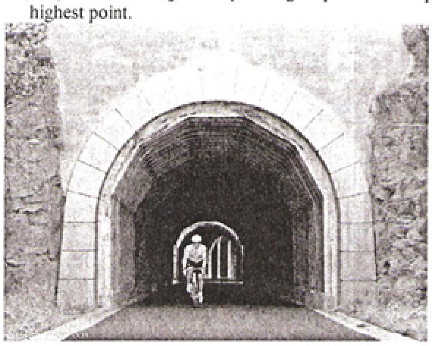
$64 - 8(k-3) > 0$

$64 - 8k + 24 > 0$

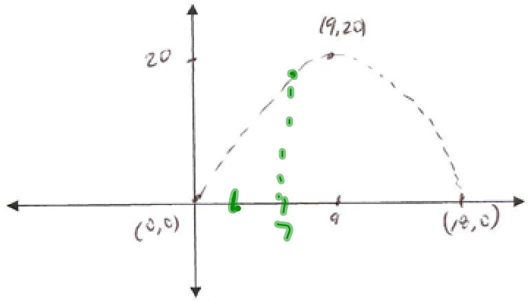
$\dfrac{-8k}{-8} > \dfrac{-88}{-8}$

$\boxed{k < 11}$

5. A cyclist is making his way through a parabolic shaped tunnel that measures 18 m wide at the base and is 20 m high at it's highest point.



(a) Draw a sketch of this tunnel on the axes below and determine a quadratic function that describes the tunnel. [4]



$y = a(x-9)^2+20$

Sub. $(0,0)$

$0 = a(0-9)^2+20$

$-20 = 81a$

$a = \frac{-20}{81}$

$$\boxed{y = \frac{-20}{81}(x-9)^2+20}$$

(b) The cyclist is situated 2 m from the centre of the tunnel and measures a height of 2.4 m while sitting on his bike. Use your function that was developed in part (a) to determine how much clearance that there is between the cyclist and the tunnel while at this instance. [3]

@ $x=7$

$y = \frac{-20}{81}(7-9)^2+20$

$y = 19.01m$

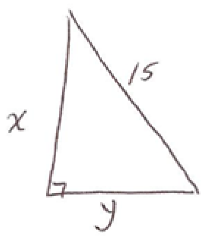
Clearance:

$= 19.01\,m - 2.4\,m$

$= 16.61\,m$

BONUS: (4 marks)

The perimeter of a right-angled triangle is 36 cm. If the hypotenuse is 15 cm, find the length of the other two sides.
(Must be done algebraically – guess and test will not be accepted!)



$x+y+15=36$

$x = 21-y$

$x^2+y^2=15^2$

$(21-y)^2+y^2 = 225$

$441-42y+y^2+y^2=225$

$2y^2-42y+441-225=0$

$2y^2-42y+216=0$

$y^2-21y+108=0$

$(y-12)(y-9)=0$

$y=12$ or $y=9$

$x=9$ $\quad x=12$

9cm & 12cm

Sample Problems – Slope and Function Notation

Slope: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

WARM-UP...

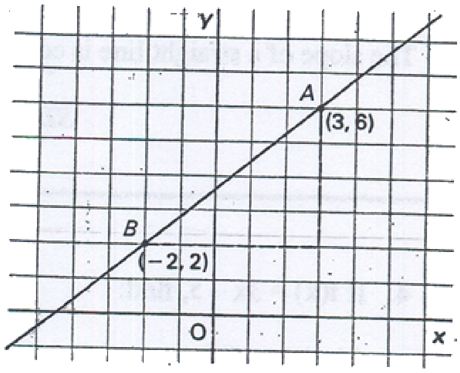
Lines that rise from left to right have a positive slope.
Lines that fall from left to right have a negative slope.

Function Notation: $f(x) = x + 3$ "f at x equals x plus three"
$h(t) = t^2 - 3t + 2$ "h at t equals t squared minus three times x plus two"

$f(6) = 6 + 3 = 9$ $h(-1) = (-1)^2 - 3(-1) + 2 = 1 + 3 + 2 = 6$

1. a) Find the slope of the line.

$M = \frac{6-2}{3-(-2)} = \frac{4}{5}$



b) Find the slope of the line containing the given points.

(i) (3, 2), (10, 14) (ii) (-6, 10), (-11, 7) (iii) (-9, -13), (1, 1)

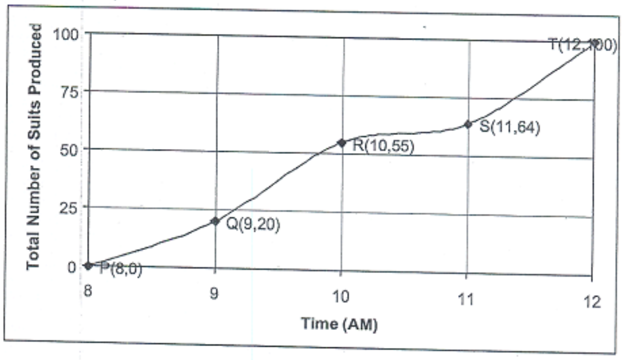
$M = \frac{10-7}{-6-(-11)} = \frac{3}{5}$

2. a) If $f(x) = 2x + 5$, find $f(-1)$. b) If $h(t) = 3t^2 - 6t + 5$, find $h(1.7)$.

3. If $g(x) = 3x^2 - 5x$, find $g(4) - g(-1)$.

Investigation 1 (altern[illegible])
Average Rate of Change

The graph above shows the total production of suits by Raggs Ltd. during one morning of work. Industrial psychologists have found curves like this typical of the production of factory workers.



1. What was the hour-by-hour production for the morning?

Time	8 - 9	9 - 10	10 - 11	11 - 12
# suits				

2. What was the average hourly production from 10AM to 12NOON?

   What is the slope of the segment RT?

3. What was the average number of suits produced per hour from 8AM to 12 NOON?

   What is the slope of PT?

4. What conjecture can you make about the average production and the slopes of the secants?

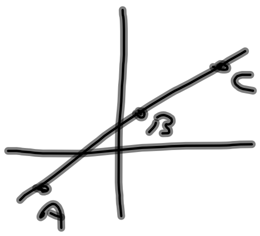
Average Rate of Change

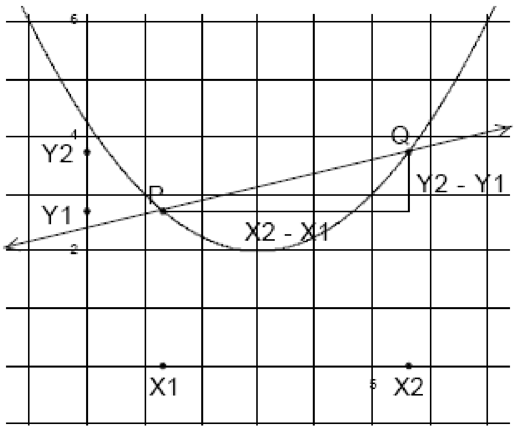
The average rate of change for a function y = f(x) is the ratio of the change in y to the change in x. Thus, if the change in x is from x_1 to x_2, the

$$\boxed{\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}}$$

$\underbrace{f(x)}=6x-3$

$y=6x-3$





- The average rate of change of a linear function is constant.
- A variable rate of change implies some kind of curve.
- Rates of change tell how one thing is changing in relation to another.
- The average rate of change of y with respect to x over an interval from a to b is

- A function with a positive slope indicates a <u>positive rate of change</u>, and hence is called an ***increasing function***. Ex: work longer hours, earn more money if being paid hourly.
- A negative slope indicates a <u>negative rate of change</u> and is called a ***decreasing function***. Ex: as height above sea level increases, air temperature decreases.
- The linear function with a <u>zero rate of change</u> is called a ***constant function***. Ex: On salary, pay remains the same despite number of hours worked.
- For <u>nonlinear functions</u>, the ***rate of change*** is not the same everywhere. It ***varies*** from place to place. Ex: speed of a baseball tossed into the air.
- The rate of change is closest to <u>zero at</u> the <u>maximum</u> or <u>minimum</u> point.