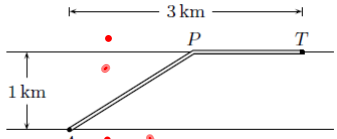
4) 13. Find the absolute extrema of $f(x) = (2x-1)\sqrt[3]{x}$ on $[-1, 1]$.

5) 14. An oil company has a refinery at point A on the bank of a straight river 1 kilometer wide. It is going to run a pipe from point A to point P somewhere on the opposite side of the river, and then straight along the river to a tank T situated 3 kilometers downstream from A. It costs 15 thousand dollars per kilometer to run the pipe under the water and 9 thousand dollars per kilometer to run the pipe along the bank. What should be the distance from P to T in order to minimize the total cost of the pipe?



$f'(x) = 2x^{1/3} + (2x-1)\left(\frac{1}{3}x^{-2/3}\right)$

$x^{-2/3}$ $\quad x^{-2/3}$

Critical Values:

$3x^{2/3}$ $\quad 3x^{2/3}$ $\quad 3x^{2/3}$

$0 = 2x^{1/3} + \frac{2x-1}{3x^{2/3}}$ $\quad x = 0$ ($f'(x)$ undefined)

$0 = 6x + 2x - 1$

Factoring

$0 = x^{-2/3}\left(2x + (2x-1)\left(\frac{1}{3}\right)\right)$

$\frac{1}{8} = \frac{8x}{8}$

$x = \frac{1}{8}$

$0 = \frac{1}{x^{2/3}}\left(2x + \frac{2}{3}x - \frac{1}{3}\right)$

x	y	
-1	3	$f(-1)$
0	0	$f(0)$
$\frac{1}{8}$	-0.375	$f(\frac{1}{8})$
1	1	$f(1)$

$x = 0$

or

$2x + \frac{2}{3}x - \frac{1}{3} = 0$

$6x + 2x - 1 = 0$

$8x = 1$

$x = \frac{1}{8}$

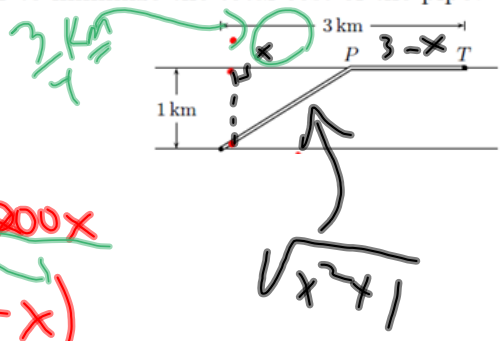
Abs. Max = 3

Abs. Min. = -0.375

$x \rightarrow$ # beyond 30

$$Rev = (30 + x)(20 - 0.5x)$$

An oil company has a refinery at point A on the bank of a straight river 1 kilometer wide. It is going to run a pipe from point A to point P somewhere on the opposite side of the river, and then straight along the river to a tank T situated 3 kilometers downstream from A. It costs 15 thousand dollars per kilometer to run the pipe under the water and 9 thousand dollars per kilometer to run the pipe along the bank. What should be the distance from P to T in order to minimize the total cost of the pipe?



$C(x) = 15000\sqrt{x^2+1} + 9000(3-x)$ $+27000 - 9000x$

$C'(x) = 15000(x^2+1)^{-\frac{1}{2}}(2x) - 9000$

$0 = \frac{15000x}{\sqrt{x^2+1}} - 9000$

$9000 = \frac{15000x}{\sqrt{x^2+1}}$

$\left(9 = \frac{15x}{\sqrt{x^2+1}}\right)^2$

$81 = \frac{225x^2}{x^2+1}$

$81x^2 + 81 = 225x^2$

$\sqrt{81} = \sqrt{144x^2}$

$9 = 12x$

$\frac{9}{12} = x$

$\frac{3}{4} = x$ $\therefore \overline{PT} = 2\frac{1}{4}\text{ km}$

BONUS

A woman at a point A on the shore of a circular lake with radius 2 miles wants to be at the point C diametrically opposite A on the other side of the lake in the shortest possible time. She can walk at the rate of 4 mi/h and row a boat at 2 mi/h. At what angle to the diameter should she row?