Given $f(x) = x^{1/3}(4+x)$,

$$f'(x) = \frac{4x+4}{3x^{2/3}} \quad \text{and} \quad f''(x) = \frac{4x-8}{9x^{5/3}}.$$

Intercepts

(a) Find and specify all intervals where f is increasing; decreasing; concave up; and concave down.

(b) Determine the coordinates of any relative extreme values and any points of inflection.

(c) Sketch a graph of f, showing all information obtained in parts (a) and (b).

Intercepts:

x-Int ($y=0$): $0 = x^{1/3}(4+x)$, $x = 0, -4$, $(0,0)$ $(-4,0)$

y-Int: $(x=0)$, $y = 0^{1/3}(4+0)$, $y = 0$, $(0,0)$

Asymptotes: None

Inc/Dec:

Critical Values: $4(x+1)$

$4x+4 = f'(x)$

$f'(x) = 0$: $4x+4=0$, $4x=-4$, $x=-1$

$f'(x)$ undefined: $3x^{2/3} = 0$, $x=0$

	4	x+1	$3x^{2/3}$	f'	f
$(-\infty,-1)$	+	−	+	−	Dec
$(-1,0)$	+	+	+	+	Inc
$(0,\infty)$	+	+	+	+	Inc

Local Max: None

Local Min: $(-1,-3)$

Concavity

Critical Values:

$f''(x) = \frac{4(x-2)}{9x^{5/3}}$

$f''(x) = 0$: $x = 2$

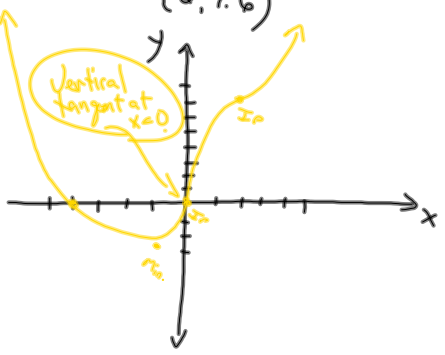
$f''(x)$ undefined: $9x^{5/3} = 0$, $x = 0$

	4	x−2	$9x^{5/3}$	f''	f
$(-\infty,0)$	+	−	−	+	Up
$(0,2)$	+	−	+	−	Down
$(2,\infty)$	+	+	+	+	Up

Inflection Points

$(0,0)$ & $(2, 6\sqrt[3]{2})$

$(2, 7.6)$



Given: $f(x) = \dfrac{18(x-1)}{x^2}$,

$f'(x) = \dfrac{-18(x-2)}{x^3}$ and $f''(x) = \dfrac{36(x-3)}{x^4}$.

(a) Find all intercepts, asymptotes, relative extrema and points of inflection of $f(x)$.

(b) Use the information in (a) to sketch a large well-labeled graph of $f(x)$.

Intercepts

x-Int: $0 = \frac{18(x-1)}{x^2}$, $x = 1$, $(1, 0)$

y-Int: $y = \frac{18(0-1)}{0^2}$, undefined (None)

Asymptotes

Vertical (Let Den = 0): $x^2 = 0$, $x = 0$

Horizontal: $\lim_{x \to \pm\infty} \frac{18x - 18}{x^2}$ "Always" $\frac{x^2}{x^2}$, $y = \frac{0 - 0}{1}$, $y = 0$

Inc/Dec

Critical Values of $f'(x)$: $\frac{-18(x-2)}{x^3} = f'(x)$, $x = 2, 0$

	-18	$x-2$	x^3	f'	f
$(-\infty, 0)$	−	−	−	−	Dec
$(0, 2)$	−	−	+	+	Inc
$(2, \infty)$	−	+	+	−	Dec

Local Max: $(2, 4.5)$

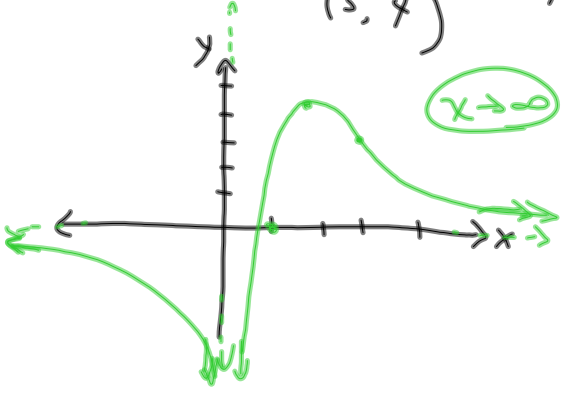
Local Min: None

Concavity

Critical Values of f'': $\frac{36(x-3)}{x^4} = f''(x)$, $x = 3, 0$

	36	$x-3$	x^4	f''	f
$(-\infty, 0)$	+	−	+	−	Dn
$(0, 3)$	+	−	+	−	Dn
$(3, \infty)$	+	+	+	+	Up

Inf. Point: $(3, 4)$



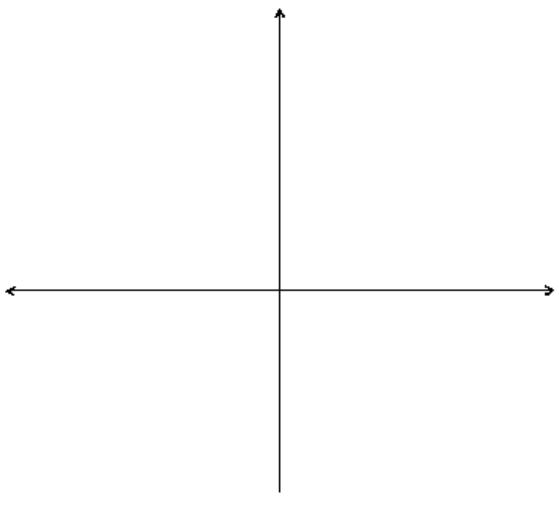
2. Consider the function : $f(x) = 10x^4 - 30x^3 + 30x^2 - 10x$ ***(value = 20)***

where $f'(x) = 10(x-1)^2(4x-1)$ and $f''(x) = -60(1-2x)(x-1)$

Supply the information requested in the boxes at right and give a careful sketch of f on the axes below.

(***Note:*** $f\left(\frac{1}{4}\right) \approx -1.1$ *and* $f\left(\frac{1}{2}\right) \approx -0.6$)

x-intercept(s)
y-intercept(s)
Region(s) of increase
Region(s) of decrease
Local maxima
Local minima
Region(s) where concave up
Region(s) where concave down
Point(s) of inflection



Calculus 120
Test : Curve Sketching

1. Consider the function : $f(x) = \dfrac{4(x^2 - x - 2)}{(x+2)^2}$ ***(value = 20)***

given $f'(x) = \dfrac{4(5x+2)}{(x+2)^3}$ and $f''(x) = \dfrac{-8(5x-2)}{(x+2)^4}$

Supply the information requested in the boxes at right and give a careful sketch of f on the axes below.

(***Note:*** $f\left(-\dfrac{2}{5}\right) \approx -2.25$ *and* $f\left(\dfrac{2}{5}\right) \approx -1.56$)

x-intercept(s)
y-intercept(s)
Vertical asymptote(s)
Horizontal asymptote(s)
Region(s) of increase
Region(s) of decrease
Local maxima
Local minima
Region(s) where concave up
Region(s) where concave down
Point(s) of inflection