2. Consider the function: $f(x)=10x^4-30x^3+30x^2-10x$ ***(value = 20)***

where $f'(x)=10(x-1)^2(4x-1)$ and $f''(x)=-60(1-2x)(x-1)$

Supply the information requested in the boxes at right and give a careful sketch of f on the axes below.

(*Note:* $f\left(\frac{1}{4}\right)\approx -1.1$ *and* $f\left(\frac{1}{2}\right)\approx -0.6$)

x-intercept(s)	$(0,0)$ $(1,0)$
y-intercept(s)	$(0,0)$
Region(s) of increase	$(\frac{1}{4},1)$ & $(1,\infty)$
Region(s) of decrease	$(-\infty,\frac{1}{4})$
Local maxima	None
Local minima	$(\frac{1}{4},-1.1)$
Region(s) where concave up	$(-\infty,\frac{1}{2})$ $(1,\infty)$
Region(s) where concave down	$(\frac{1}{2},1)$
Point(s) of inflection	$(\frac{1}{2},-0.6)$ $(1,0)$

X-Int:

$0=10x^4-30x^3+30x^2-10x$

$0=10x(x^3-3x^2+3x-1)$

$x=1$: $1-3+3-1=0$

$\therefore (x-1)$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1 \\ & & 1 & -2 & 1 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$10x(x-1)(x^2-2x+1)=0$

$10x(x-1)(x-1)^2$

$10x(x-1)^3=0$

$x=0,1$

y-Int

$(0,0)$

Critical Values of f'

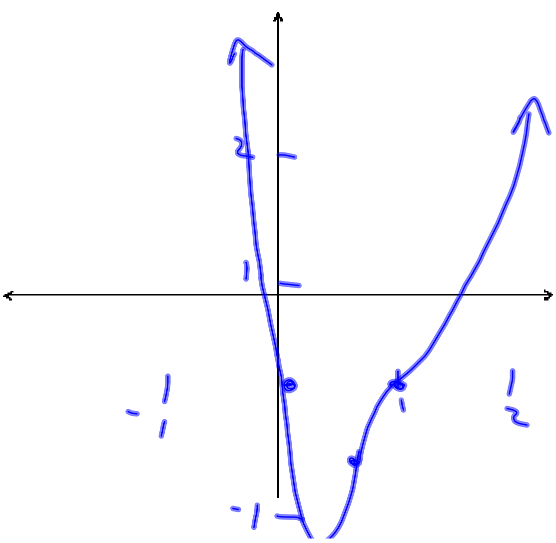
$x=1,\frac{1}{4}$

	10	$(x-1)^2$	$4x-1$	f'	f
$(-\infty,\frac{1}{4})$	+	+	−	−	Dec
$(\frac{1}{4},1)$	+	+	+	+	Inc
$(1,\infty)$	+	+	+	+	Inc

Critical Values of f''

$x=\frac{1}{2},1$

	-60	$1-2x$	$x-1$	f''	f
$(-\infty,\frac{1}{2})$	−	+	−	+	Up
$(\frac{1}{2},1)$	−	−	−	−	Down
$(1,\infty)$	−	−	+	+	Up



Calculus 120
Test : Curve Sketching

1. Consider the function : $f(x)=\dfrac{4\left(x^2-x-2\right)}{(x+2)^2}$ **(value = 20)**

given $f'(x)=\dfrac{4(5x+2)}{(x+2)^3}$ and $f''(x)=\dfrac{-8(5x-2)}{(x+2)^4}$

Supply the information requested in the boxes at right and give a careful sketch of $\boldsymbol{f}$ on the axes below.

(***Note:*** $f\left(-\frac{2}{5}\right)\approx -2.25$ *and* $f\left(\frac{2}{5}\right)\approx -1.56$)

x-intercept(s)	$(2,0)$ $(-1,0)$
y-intercept(s)	$(0,-2)$
Vertical asymptote(s)	$x=-2$
Horizontal asymptote(s)	$y=4$
Region(s) of increase	$(-\infty,-2)$ $(-\frac{2}{5},\infty)$
Region(s) of decrease	$(-2,-\frac{2}{5})$
Local maxima	None
Local minima	$(-\frac{2}{5},-2.25)$
Region(s) where concave up	$(-\infty,\frac{2}{5})$
Region(s) where concave down	$(\frac{2}{5},\infty)$
Point(s) of inflection	$(\frac{2}{5},-1.56)$

X-Int:
$x^2-x-2=0$
$(x-2)(x+1)=0$
$x=2,-1$

Y-Int.
$y=\dfrac{4(0-0-2)}{\cancel{(0+2)^2}}$
$y=-2$

Vertical
$(x+2)^2=0$
$x=-2$

Horizontal
$\lim\limits_{x\to\pm\infty}\dfrac{\frac{4x^2}{x^2}-\frac{4x}{x^2}-\frac{8}{x^2}}{\frac{x^2}{x^2}+\frac{4x}{x^2}+\frac{4}{x^2}}=4$

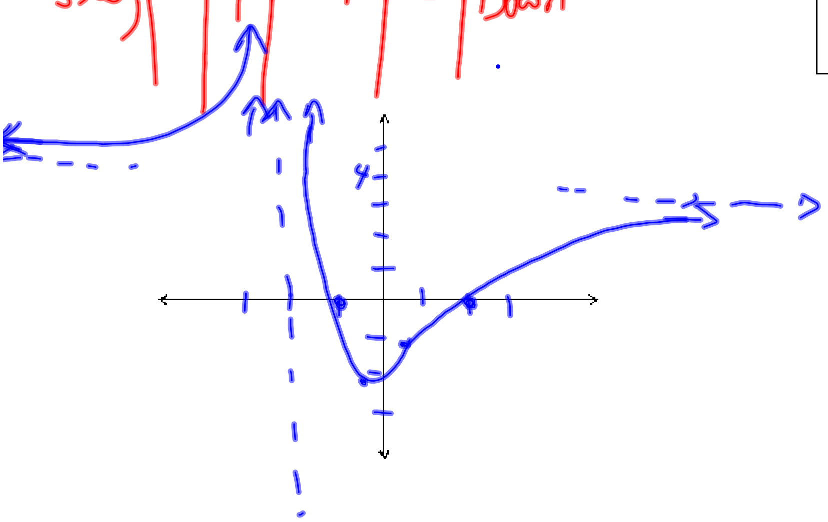
$y=4$

Critical Values of f'
$x=-\frac{2}{5},-2$

	4	5x+2	$(x+2)^3$	f'	f
$(-\infty,-2)$	+	−	−	+	Inc
$(-2,-\frac{2}{5})$	+	−	+	−	Dec
$(-\frac{2}{5},\infty)$	+	+	+	+	Inc

Critical Values of f''
$x=\frac{2}{5},-2$

	−8	5x−2	$(x+2)^4$	f''	f
$(-\infty,-2)$	−	−	+	+	Up
$(-2,\frac{2}{5})$	−	−	+	+	Up
$(\frac{2}{5},\infty)$	−	+	+	−	Down



Curve Sketching: Practice Questions

Page 240 from your textbook...

2. $y=(x^2-1)^3$ $\quad y'=6x(x^2-1)^2$ $\quad y''=6(x^2-1)(5x^2-1)$

4. $y=\dfrac{x^2}{x^2+3}$ $\quad y'=\dfrac{6x}{(x^2+3)^2}$ $\quad y''=\dfrac{18(1-x^2)}{(x^2+3)^3}$

5. $y=\dfrac{x}{x^2-1}$ $\quad y'=\dfrac{-x^2-1}{(x^2-1)^2}$ $\quad y''=\dfrac{2x^3+6x}{(x^2-1)^3}$

8. $y=\dfrac{x^2-1}{x^3}$ $\quad y'=\dfrac{-(x^2-3)}{x^4}$ $\quad y''=\dfrac{2(x^2-6)}{x^5}$

$$F(x) = \frac{(x+2)^2}{x^2+4}$$

$$F'(x) = \frac{(16-4x^2)}{(x^2+4)^2}$$

$$F''(x) = \frac{8x(x^2+12)}{(x^2+4)^3}$$

$\boxed{\begin{array}{l} y\text{-int} = 1 \\ x\text{-int} = -2 \end{array}}$

H-asymptote: $\boxed{y=1}$

V-asymptote:

6. Consider the graph of the function (UNB 2000 Final Exam)

$$f(x) = \frac{(x+2)^2}{x^2+4}.$$

You are given that

$$f'(x) = \frac{16-4x^2}{(x^2+4)^2} \quad \text{and} \quad f''(x) = \frac{8x(x^2-12)}{(x^2+4)^3}.$$

(a) Find the x-intercept.

(b) Find all vertical or horizontal asymptotes, if any.

(c) Determine the intervals where $f(x)$ is increasing or decreasing.

(d) Determine the intervals where $f(x)$ is concave up or concave down.

(e) Find all inflection points.

(f) Find the absolute maximum value of $f(x)$, if it exists.

(g) Sketch the graph of $f(x)$.

Calculus 120
Test : Curve Sketching

1. Consider the function : $f(x) = \frac{8(x-2)}{x^2}$ ***(value = 20)***

given $f'(x) = \frac{-8(x-4)}{x^3}$ and $f''(x) = \frac{16(x-6)}{x^4}$

Supply the information requested in the boxes at right and give a careful sketch of f on the axes below.

X-Int. $(y=0)$

$0 = \frac{8(x-2)}{x^2}$

$x = 2$

y-Int. $(x=0)$

$\frac{8(0-2)}{0^2}$ = undefined

$\therefore$ None

Asymptotes.

Vertical (Set den. = 0)

$x^2 = 0$

$x = 0$

Horizontal $(x \to \pm\infty)$

$\lim_{x \to \pm\infty} \frac{\frac{8x}{x^2} - \frac{16}{x^2}}{\frac{x^2}{x^2}} = \frac{0}{1}$

$y = 0$

Inc/Dec

$\frac{-8(x-4)}{x^3} = 0$

Critical Values:

$x = 4, 0$

	-8	$x-4$	x^3	f'	f
$(-\infty,0)$	−	−	−	−	Dec
$(0,4)$	−	−	+	+	Inc
$(4,\infty)$	−	+	+	−	Dec

Concavity

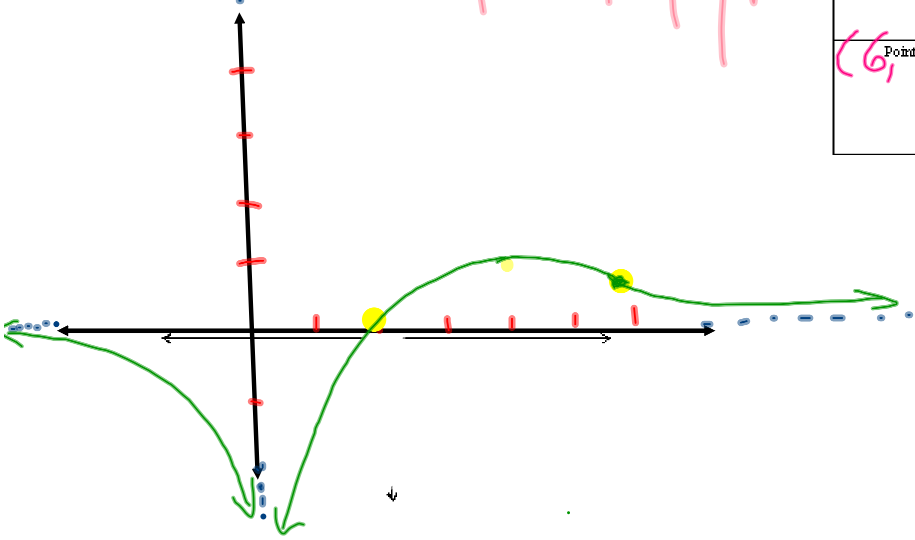
$\frac{16(x-6)}{x^4} = 0$

Critical Values:

$x = 6, 0$

	16	$x-6$	x^4	f''	f
$(-\infty,0)$	+	−	+	−	Down
$(0,6)$	+	−	+	−	Down
$(6,\infty)$	+	+	+	+	Up

x-intercept(s)	$(2,0)$
y-intercept(s)	None
Vertical asymptote(s)	$x = 0$
Horizontal asymptote(s)	$y = 0$
Region(s) of increase	$(0,4)$
Region(s) of decrease	$(-\infty,0)$ & $(4,\infty)$
Local maxima	$(4,1)$
Local minima	None
Region(s) where concave up	$(6,\infty)$
Region(s) where concave down	$(-\infty,0)$ $(0,6)$
Point(s) of inflection	$(6, \frac{8}{9})$



$f(x) = x^4 - 2x^2$ $\quad f'(x) = 4x^3 - 4x = 4x(x^2-1)$ $\quad f''(x) = 12x^2 - 4$

2. Consider the function: $f(x) = x^4 - 2x^2$ **(value = 20)**

Supply the information requested in the boxes at right and provide a careful sketch of f on the axes below.

Note: $f\left(\pm\frac{1}{\sqrt{3}}\right) \approx -\frac{5}{9}$

x-intercept(s)	$(0,0)$ $(\sqrt{2},0)$ $(-\sqrt{2},0)$
y-intercept(s)	$(0,0)$
Region(s) of increase	$(-1,0)$ & $(1,\infty)$
Region(s) of decrease	$(-\infty,-1)$ & $(0,1)$
Local maxima	$(0,0)$
Local minima	$(-1,-1)$ $(1,-1)$
Region(s) where concave up	$(-\infty,-\frac{1}{\sqrt{3}})$ & $(\frac{1}{\sqrt{3}},\infty)$
Region(s) where concave down	$(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})$
Point(s) of inflection	$(-\frac{1}{\sqrt{3}},-0.6)$ $(\frac{1}{\sqrt{3}},-0.6)$

≈ -0.5

Intercepts

x-Int ($y=0$)

$0 = x^4 - 2x^2$

$0 = x^2(x^2-2)$

$0 = x^2(x-\sqrt{2})(x+\sqrt{2})$

$x = 0, \pm\sqrt{2}$

y-Int. ($x=0$)

$y = 0^2 - 0$

$y = 0$

Inc/Dec

$4x(x^2-1) = 0$

$4x(x-1)(x+1) = 0$

$x = 0, \pm 1$

	$4x$	$x-1$	$x+1$	f'	f
$(-\infty,-1)$	−	−	−	−	Dec
$(-1,0)$	−	−	+	+	Inc
$(0,1)$	+	−	+	−	Dec
$(1,\infty)$	+	+	+	+	Inc

Concavity

$12x^2 - 4 = 0$

$4(3x^2-1) = 0$

$4(\sqrt{3}x-1)(\sqrt{3}x+1)$

$x = \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$

	4	$\sqrt{3}x-1$	$\sqrt{3}x+1$	f''	f
$(-\infty,-\frac{1}{\sqrt{3}})$	+	−	−	+	up
$(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})$	+	−	+	−	down
$(\frac{1}{\sqrt{3}},\infty)$	+	+	+	+	up