(8) 13. Santa Claus has a piece of land on which he would like to grow apples. Research has shown that if he plants 24 trees, each tree will produce 600 apples per year. For each additional tree planted, the number of apples on each tree will decrease by 12 apples per year. How many trees should Santa plant to maximize his apple production?

Be sure to justify your work. UNB: Winter 2009

6. Find the absolute minimum value of Penn University: Fall 2011

$$f(x) = x - \frac{4x}{x+1}$$

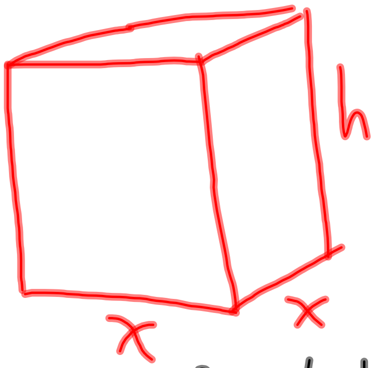
on the interval $[0, 3]$

A) 0 B) 1 C) -1 D) 2

E) -2 F) $\frac{1}{2}$ G) $-\frac{1}{2}$ H) 3

900 square centimeters of material is to be used to make an open-topped box which has a square base. What should be the dimensions of the box in order that its volume will be as large as possible?

Binghamton University: 2010 Final



$V = x^2 h$

$x^2 + 4xh = 900$

$$\boxed{h = \frac{900 - x^2}{4x}}$$

$V = x^2\left(\frac{900 - x^2}{4x}\right)$

$V = \frac{900x^2}{4x} - \frac{x^4}{4x}$

$V = 225x - \frac{1}{4}x^3$

$V' = 225 - \frac{3}{4}x^2$

$0 = 225 - \frac{3}{4}x^2$

$(4)\ \frac{3}{4}x^2 = 225(4)$

$\frac{3x^2}{3} = \frac{900}{3}$

$x^2 = 300$

$x = \sqrt{300}$

$x = 17.3$

$h = \frac{900 - (\sqrt{300})^2}{4\sqrt{300}}$

$h = \frac{600}{4\sqrt{300}} = 8.7$

Dimensions: $\underline{17.3\text{cm} \times 17.3\text{cm} \times 8.7\text{cm}}$

Let $f(x) = 2 - 2x - x^3$.

(a) What is the domain of f?

(b) Where does its graph cross the y-axis? (Don't try to calculate where it crosses the x-axis.)

(c) On what intervals is f increasing? (if none say so).

(d) On what intervals is f decreasing? (if none say so).

(e) Find the local minima of f (if any).

(f) Find the local maxima of f (if any).

Binghamton University: 2009 Final

$f'(x) = -2 - 3x^2$

Critical Values:

$0 = -2 - 3x^2$

$2 = -3x^2$

$-\frac{2}{3} = x^2$

Impossible... No critical Values

(a) $x \in \mathbb{R}$

(b) Let $x = 0$

$y = 2 - 0 - 0$

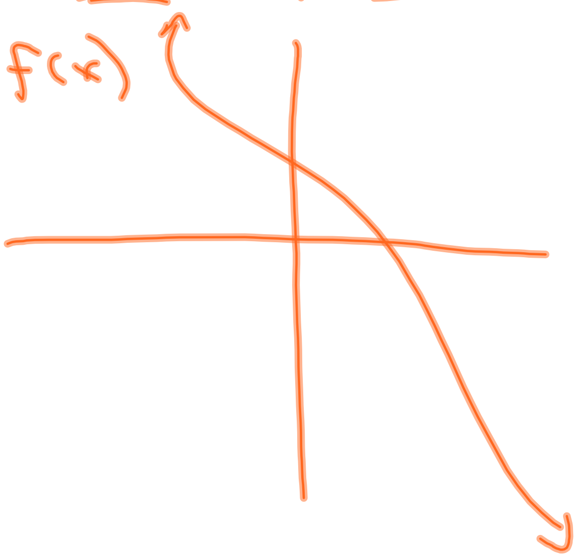
$y = 2$

(d) No critical values... $\therefore$ always Inc OR Dec??

$f'(x) = -2 - 3x^2 < 0$

$\therefore f(x)$ is decreasing $(-\infty, \infty)$

(c) None (e) None (f) None



(a) Let

$$f(x) = x^3(x-1)^2$$

Find all relative extrema of f. — Sign Table

(b) Find the absolute maximum and minimum of $f(x) = x^3 - 6x + 1$ on the interval $[-2, 0]$.

Howard University: Final 09

common factors

(a) $f'(x) = 3x^2(x-1)^2 + x^3(2(x-1))$

$0 = x^2(x-1)[3(x-1) + x(2)]$

$0 = x^2(x-1)(3x-3+2x)$

$0 = x^2(x-1)(5x-3)$

$x = 0, 1, 3/5$

	x^2	$x-1$	$5x-3$	f'	f
$(-\infty, 0)$	+	−	−	+	Inc
$(0, 3/5)$	+	−	−	+	Inc
$(3/5, 1)$	+	−	+	−	Dec
$(1, \infty)$	+	+	+	+	Inc

Local Max

Local Min

Local Max: $\left(\frac{3}{5}, 0.03\right)$

Local Min. $(1, 0)$

(b) $f(x) = x^3 - 6x + 1$ $[-2, 0]$

$f'(x) = 3x^2 - 6$

$0 = 3x^2 - 6$

$6 = 3x^2$

$\sqrt{2} = \sqrt{x^2}$

$\pm\sqrt{2} = x$

$x = \cancel{\sqrt{2}}$ or $-\sqrt{2}$

x	y
-2	5
$-\sqrt{2}$	6.65
0	1

Abs. Max = 6.65

Abs. Min. = 1

A cylindrical can is to have a volume of 1200 cm^3. The material for the side of the can costs 2 cents per cm^2 and the material for the top and bottom of the can costs 3 cents per cm^2. Find the radius of the base of the can which minimizes the cost.

Howard University: Final 09

$A = \pi r^2$ $\quad V = \pi r^2 h$

$A = 2\pi r h$

$V = 1200$

$\pi r^2 h = 1200$

$h = \dfrac{1200}{\pi r^2}$

Top & Bottom

$\text{Cost} = 3(2\pi r^2) + 2(2\pi r h)$

$C = 6\pi r^2 + 4\pi r h$

$C = 6\pi r^2 + 4\pi r\left(\dfrac{1200}{\pi r^2}\right)$

$C = 6\pi r^2 + 4800\, r^{-1}$

$C' = 12\pi r - 4800 r^{-2}$

$0 = 12\pi r - \dfrac{4800}{r^2}$

$0 = 12\pi r^3 - 4800$

$\dfrac{4800}{12\pi} = \dfrac{12\pi r^3}{12\pi}$

$r = \sqrt[3]{\dfrac{4800}{12\pi}}$

$r = \underline{5.03 \text{ cm}}$

$h = \dfrac{1200}{\pi (5.03)^2}$

$h = \underline{15.1 \text{ cm}}$