2005 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

2. The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1+3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

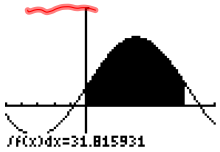
(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

(a) $\int_0^6 \left(2 + 5\sin\left(\frac{4\pi t}{25}\right)\right) dt$



$= 2t - \frac{125}{4\pi}\cos\left(\frac{4\pi t}{25}\right)\Big|_0^6$

$= \left[12 - \frac{125}{4\pi}\cos\left(\frac{24\pi}{25}\right)\right] - \left[0 - \frac{125}{4\pi}\right]$

=

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24π/25)+125/(4π)
      2.825466339
12-125/(4π)cos((
24π/25))+125/(4π
)
      31.81593137
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(b) $Y(t) = 2500 + \int_0^t \left[\left(\frac{15t}{1+3t}\right) - \left(2 + 5\sin\left(\frac{4\pi t}{25}\right)\right)\right] dt$

(c) $S(t) - R(t)$ @ $t = 4$

$\left(\frac{15(4)}{1+12}\right) - \left(2 + 5\sin\left(\frac{4\pi(4)}{25}\right)\right)$

$= -1.909 \text{ yd}^3/\text{hr}.$

(d) $Y(t) = 2500 + \int_0^t \left[\left(\frac{15t}{1+3t}\right) - \left(2 + 5\sin\left(\frac{4\pi t}{25}\right)\right)\right] dt$

Amount of Sand to be Minimum

$\frac{.5t}{1+3t} - 2 - 5\sin\frac{4\pi t}{25} = 0$