Warm-up...

Change base and evaluate:

a) $\log_4 25$

$$\frac{\log 25}{\log 4} = 2.32$$

b) $\log_2 46$

$$\frac{\log 46}{\log 2} = 5.52$$

Write the following as a single logarithm...

$$\frac{1}{4}\left[2\left(\log_2 x + 3\log_2 y\right) - 3\log_2 z\right]$$

Solve the following... $\dfrac{3^x \bullet 2^{x+1}}{4^{3x}} = 5^{2x}$

Write the following as a single logarithm...

$$\frac{1}{4}\left[2\left(\log_2 x + 3\log_2 y\right) - 3\log_2 z\right]$$

$$\frac{1}{4}\left[\left(\log_2 x + 3\log_2 y\right)^2 - \log_2 z^3\right]$$

$$\frac{1}{4}\left\{\log_2 x^2 + \log_2 y^6 - \log_2 z^3\right]$$

i)

$$\frac{1}{4}\left[\log_2 \frac{x^2 y^6}{z^3}\right]$$

ii)

$$\left(\log_2 \frac{x^2 y^6}{z^3}\right)^{1/4}$$

iii)

$$\log_2 x^{1/2} y^{3/2} z^{3/4}$$

Solve the following... $\dfrac{3^x \bullet 2^{x+1}}{4^{3x}} = 5^{2x}$ — x=0.1241

$$\log\left(\frac{3^x \cdot 2^{x+1}}{4^{3x}}\right) = \log 5^{2x}$$

$$\log 3^x + \log 2^{x+1} - \log 4^{3x} = \log 5^{2x}$$

$$(x)\log 3 + (x+1)\log 2 - (3x)\log 4 = (2x)\log 5$$

$$\log 3(x) + \log 2(x) + \log 2 - 3\log 4(x) = 2\log 5(x)$$

$$\log 3(x) + \log 2(x) - 3\log 4(x) - 2\log 5(x) = -\log 2$$

$$x(\log 3 + \log 2 - 3\log 4 - 2\log 5) = -\log 2$$

$$(\log 3 + \log 2 - 3\log 4 - 2\log 5)$$

$$x = \frac{-\log 2}{\log 3 + \log 2 - 3\log 4 - 2\log 5}$$

0.12
0.736
-7.5

Applications of Logarithms

A $1,000 deposit is made at a bank that pays 12% compounded annually. How much will you have in your account at the end of 10 years?

x	0	1	2
y	1000	1120	1254.4

$$y = 1000(1.12)^x$$

$$y = 1000(1.12)^{10}$$

$$= \$3105.84$$

Now you Try!

#1. An investment of $100 is made in a term deposit that pays 9.5 % / year compounded annually. How long will it take to triple the value of the investment?

$$y = 100(1.095)^x$$

$$\frac{300}{100} = \frac{100(1.095)^x}{100}$$

$$3 = (1.095)^x$$

$$\log 3 = \log(1.095)^x$$

$$\log 3 = x \log(1.095)$$

$$x = \frac{\log 3}{\log(1.095)}$$

$$\boxed{x = 12.1 \text{ years}}$$

Applications of Logarithms

Hospitals utilize the radioactive substance iodine-131 in the diagnosis of conditions of the thyroid gland. The half-life of iodine-131 is eight days.

a) If a hospital acquires 2 g of iodine-131, how much of this sample will remain after 20 days ?

b) How long will it be until only 0.01 g remains?

x	0	8	16
y	2	1	0.5

$$y = 2(0.5)^{\frac{x}{8}}$$

$$y = 2(0.5)^{\frac{20}{8}}$$

b) $y = 2(0.5)^{\frac{x}{8}}$ $y = 0.35 g$

$$\frac{0.01}{2} = \frac{2(0.5)^{\frac{x}{8}}}{2}$$

$$0.005 = (0.5)^{\frac{x}{8}}$$

$$\log(0.005) = \log(0.5)^{\frac{x}{8}}$$

$$\frac{\log(0.005)}{\log(0.5)} = \frac{\frac{x}{8}\log(0.5)}{\log(0.5)}$$

$$\frac{7.64}{.8} = \frac{x \cdot 8}{8}$$

$$61.15 = x$$

Now you Try!

If you start a biology experiment with 5,000,000 cells and 45% of the cells are dying every minute, how long will it take to have 1,000 cells?

$$y = 5000000(0.45)^x$$

$$\frac{1000}{5000000} = \frac{5000000(0.45)^x}{5000000}$$

$$2 \times 10^{-4} = (0.45)^x$$

$$\frac{\log(2 \times 10^{-4})}{\log .45} = \frac{x \log(0.45)}{\log(0.45)}$$

$$x = -11.7$$

#2. Mr. Svarc has been growing a new bacteria culture. The population was intially 5000 bacteria and doubles every three hours.

a) What will be the population after 240 minutes.

$= 4 \text{hrs}$

x	0	3
y	5000	10000

$$y = 5000(2)^{x/3}$$

$$y = 5000(2)^{4/3}$$

$$y = 12599.2$$

b) How many hours will it take for the bacteria population to reach 1 million?

$$1000000 = 5000(2)^{x/3}$$

HOMEWORK: Work on the following...

Page 177:   #4
             #5

Page 181:   #11 (1st column)
             #12
             #14

Page 183:   #24
             #25(1st column)

Answers

4. (a) 2.68   (b) 1.59   (c) 2.98   (d) 1.85   (e) 1.87   (f) 1.26

QUESTION 5   Page 178

If students understand the "Think about ... Question 5" in the margin and perform this operation of changing from logarithmic to exponential form, then they should realize that the resulting equations are similar to the ones they encountered in Question 4.

Answers

5. (a) 1.85   (b) 0.73

Answers

11. (a) 2.585   (b) 0.811   (c) 1.200   (d) −5.213
    (e) 0.132   (f) 1.701   (g) 1.461   (h) 1.113

Answers

12. (a) 2   (b) 25   (c) 2   (d) 16
    (e) 196   (f) $\frac{729}{64}$ (g) 5   (h) 2

Answers

14. (a) 1.796   (b) 2.377   (c) −0.115

Answers

24. (a) 4   (b) $\frac{7}{6}$ (c) 4
    (d) 0.429   (e) 2   (f) 1.661

QUESTION 25   Page 183

Answers

25. (a) 5   (b) 2 or 0   (c) 0.404   (d) $\frac{1}{7}$ or 343
    (e) 3   (f) 5   (g) $\frac{17}{14}$ (h) 0.480
    (i) 3 (the other root, −2, is extraneous)   (j) −13 or −1