Review

1. Solve: $2 \cdot 6^{x+1} = 18$.

(A) $\dfrac{\log 9 - 1}{\log 6}$

(B) $\dfrac{\log 9}{\log 6} - 1$

(C) $\dfrac{\log 18}{\log 12} - 1$

(D) $\dfrac{\log 18 - 1}{\log 12}$

2. Which is equivalent to $3\left[\frac{1}{2}\log_4 A + \log_4 B - \log_4 C\right]$?

(A) $\log_4 \dfrac{\left(B\sqrt{A}\right)^3}{C}$

(B) $\log_4 \dfrac{AB}{C}$

(C) $\log_4 \left(\sqrt{\dfrac{AB}{C}}\right)^3$

(D) $\log_4 \left(\dfrac{B\sqrt{A}}{C}\right)^3$

3. Algebraically solve: $\log_2(x+2) + \frac{1}{3}\log_2(8x^3) = 16^{\frac{1}{2}}$.

$\log_2(x+2) + \log_2(8x^3)^{1/3} = 4$

$\log_2(x+2) + \log_2(2x) = 4$

$\log_2[(x+2)(2x)] = 4$

$2^4 = 2x^2 + 4x$

$0 = 2x^2 + 4x - 16$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$x = -4, 2$ (extraneous root)

4. A truck is purchased for \$51 000 and depreciates by 23% annually. At the same time a minivan is purchased for \$38 000 and depreciates by 17% annually. Write an equation to model this situation and use it to determine when the two vehicles will be of equal value.

Truck

t	0	1
V	51000	39270

$\times 0.77$

$V = 51000(0.77)^t$

Mini Van

t	0	1
V	38000	

$\times 0.83$

$V = 38000(0.83)^t$

$\dfrac{51000(0.77)^t}{38000} = \dfrac{38000(0.83)^t}{38000}$

$\frac{51}{38}(0.77)^t = 0.83^t$

$\log\left[\frac{51}{38}(0.77)^t\right] = \log 0.83^t$

$\log \frac{51}{38} + \log 0.77^t = \log 0.83^t$

$\log \frac{51}{38} + t\log 0.77 = t\log 0.83$

$\log \frac{51}{38} = t\log 0.83 - t\log 0.77$

$\dfrac{\log \frac{51}{38}}{\log 0.83 - \log 0.77} = \dfrac{t(\log 0.83 - \log 0.77)}{\log 0.83 - \log 0.77}$

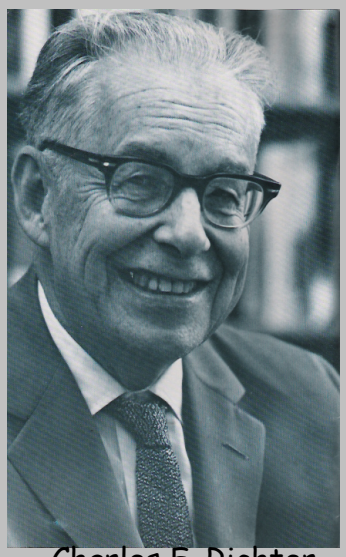
$t =$ log(51/38)/(log(.83)-log(.77))
3.921353286

Logarithmic Scales

I. Richter Scale: Severity of Earthquakes

$$R = \log_{10}\left(\frac{I}{I_o}\right)$$

R - magnitude of an earthquake
I - Intensity of the earthquake
(amplitude of the wave on a seismograph)
I_o - Intensity of the reference earthquake
(1 micron) (1 micron = 10^{-4} cm)



Charles F. Richter
1900-1985

The intensity of the earthquake is measured by the amplitude of a seismograph reading taken 100 km from the epicenter of the earthquake.

This implies that an earthquake that reads a 5 on the Richter scale would be 10 times more intense than an earthquake that reads a 4 on the Richter Scale. (The scale jumps by powers of 10)

How many times more intense would an earthquake that reads an 8 on the Richter Scale be than a 5 on the Richter Scale?

If the intensity of earthquake A is 5 and the intensity of earthquake B is 650, what is the difference in their magnitudes as measured by the Richter Scale?

2.11

$R = \log\left(\frac{I}{I_0}\right)$

$R = \log\left(\frac{5}{10^{-4}}\right)$

$R = 4.7$

$R = \log\left(\frac{650}{10^{-4}}\right)$

$R = 6.8$

The 1985 Mexico City earthquake had a magnitude of 8.1 on the Richter scale and the 1976 Tangshan earthquake was 1.26 as intense. What was the magnitude of the Tangshan earthquake on the Richter Scale?

$8.1 = \log\left(\frac{I}{10^{-4}}\right)$

$10^{8.1} = \frac{I}{10^{-4}}$

$I = (10^{8.1})(10^{-4})$

$I = 10^{4.1}$

$R = \log\left(\frac{I}{I_0}\right)$

Tangshan:

$I = 1.26 \times 10^{4.1}$

$R = \log\left(\frac{1.26(10)^{4.1}}{10^{-4}}\right)$

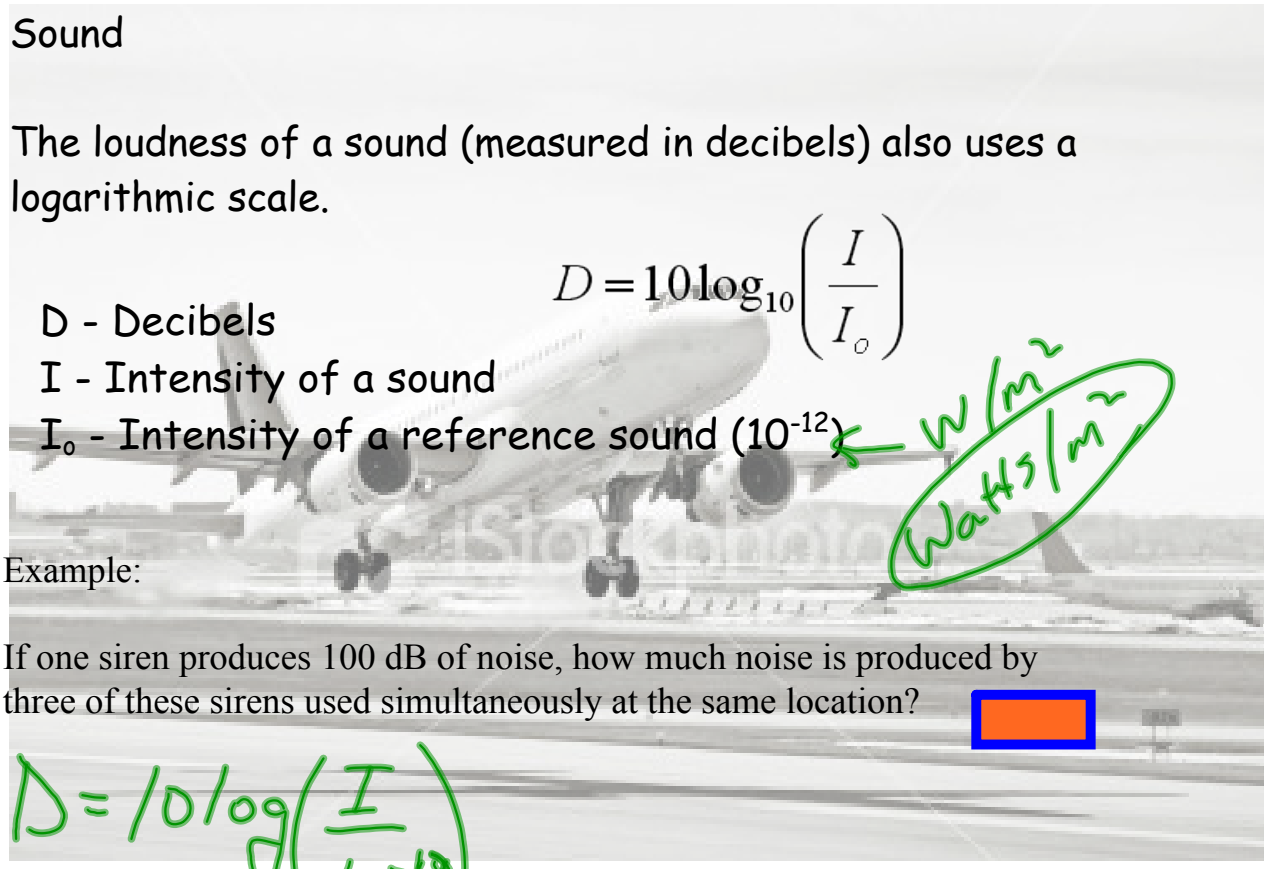
$R = 8.2$

Sound

The loudness of a sound (measured in decibels) also uses a logarithmic scale.

$$D = 10\log_{10}\left(\frac{I}{I_o}\right)$$

D - Decibels
I - Intensity of a sound
I_o - Intensity of a reference sound (10^{-12})



W/m^2 Watts/m^2

Example:

If one siren produces 100 dB of noise, how much noise is produced by three of these sirens used simultaneously at the same location?

$$D = 10\log\left(\frac{I}{10^{-12}}\right)$$

$$\frac{100}{10} = \frac{10}{10}\log\frac{I}{10^{-12}}$$

$$10 = \log\left(\frac{I}{10^{-12}}\right)$$

$$10^{10} = \frac{I}{10^{-12}}$$

$$I = (10^{10})(10^{-12})$$

$$I = 10^{-2}$$

3 sirens $\rightarrow 3 \times 10^{-2}\ W/m^2$

$$D = 10\log\left(\frac{3(10)^{-2}}{10^{-12}}\right)$$

$D = 105$ dB

Practice problems...

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