

Warm Up

a) $\left(\frac{1}{2}\right)^x = 3$

$$\log_{10}\left(\frac{1}{2}\right)^x = \log_{10} 3$$

$$x \log_{10}\left(\frac{1}{2}\right) = \log_{10} 3$$

$$\frac{\log_{10}\left(\frac{1}{2}\right)}{\log_{10}\left(\frac{1}{2}\right)} \frac{\log_{10} 3}{\log_{10}\left(\frac{1}{2}\right)}$$

$$x = -1.58$$

b) $\frac{6(2)^{x+3}}{6} = \frac{30}{6}$

$$2^{x+3} = 5$$

$$\log_{10} 2^{x+3} = \log_{10} 5$$

$$\frac{(x+3) \log_{10} 2}{\log_{10} 2} = \frac{\log_{10} 5}{\log_{10} 2}$$

$$x+3 = \left(\frac{\log_{10} 5}{\log_{10} 2}\right) - 3$$

$$x = \left(\frac{\log_{10} 5}{\log_{10} 2}\right) - 3$$

$$= -0.67$$

c)

$$4^{x+2} - 2 = 10$$

$$4^{x+2} = 12$$

$$\log 4^{x+2} = \log 12$$

$$\frac{(x+2) \log 4}{\log 4} = \frac{\log 12}{\log 4}$$

$$x+2 = \left(\frac{\log 12}{\log 4} \right) - 2$$

$$x = \left(\frac{\log 12}{\log 4} \right) - 2$$

$$= -0.208$$

From Yesterday

$$\frac{3(2^{x-4})}{3} = \frac{360}{3}$$

$$2^{x-4} = 120$$

$$\log 2^{x-4} = \log 120$$

$$(x-4)\log 2 = \log 120$$

$$x-4 = \frac{\log 120}{\log 2}$$

$$x = \left(\frac{\log 120}{\log 2} \right) + 4$$

$$x = 10.9$$

$$10^{x+5} - 8 = 60$$

$$10^{x+5} = 68$$

$$\log_{10} 10^{x+5} = \log_{10} 68$$

$$(x+5)\log_{10} 10 = \log_{10} 68$$

$$x+5(1) = \log_{10} 68$$

$$x = (\log_{10} 68) - 5$$

$$x = -3.16$$

From Yesterday

1.71
3
-1.5

$$\frac{4000 \cdot \cancel{(2+7^{2x})}}{\cancel{2+7^{2x}}} = 5 \cdot (2+7^{2x})$$

$$\frac{4000}{5} = \frac{5(2+7^{2x})}{5}$$

$$800 = 2 + 7^{2x}$$

$$798 = 7^{2x}$$

$$\log 798 = \log 7^{2x}$$

$$\frac{\log 798}{\log 7} = \frac{2x(\log 7)}{\log 7}$$

$$2x = \left(\frac{\log 798}{\log 7} \right)$$

$$x = 1.71$$

This process leads to a very useful formula...

Have you ever thought...

"b raised to what power will give me some number N?"

ie. $\log_b N = x$ or $b^x = N$

Solve the equation: $b^x = N$

Could we have taken the logarithm of each side to any base we had chosen?

This leads to the **change of base formula**:

$$\log_b N = \frac{\log_a N}{\log_a b}$$
$$= \frac{\log_7 N}{\log_7 b}$$

Evaluate each of the following:

$$\log_5 45$$
$$= \frac{\log_{10} 45}{\log_{10} 5}$$
$$= 2.36$$

$$\log_3 7$$
$$= \frac{\log_{10} 7}{\log_{10} 3}$$
$$= 1.77$$

Examples...

(1) $\log_9 75$

$$\frac{\log_{10} 75}{\log_{10} 9}$$

$$= 1.96$$

(2) $3(4)^{x-1} = 24$

$$\frac{\ln 75}{\ln 9}$$

(3) $6^{3x} = 2^{2x-3}$

Example: $\frac{2^{4x}}{5^{2x+5}} = 7^{x-1}$

$$\log_{10} \left(\frac{2^{4x}}{5^{2x+5}} \right) = \log 7^{x-1}$$

$$\log 2^{4x} - \log 5^{2x+5} = \log 7^{x-1}$$

$$(4x) \log 2 - (2x+5) \log 5 = (x-1) \log 7$$

$$(4 \log 2)x - (2 \log 5)x - 5 \log 5 = (\log 7)x - \log 7$$

$$(4 \log 2)x - (2 \log 5)x - (\log 7)x = -\log 7 + 5 \log 5$$

$$x [(4 \log 2) - (2 \log 5) - (\log 7)] = -\log 7 + 5 \log 5$$

$$[(4 \log 2) - (2 \log 5) - (\log 7)] \quad \text{" "}$$

$$x = -1.550$$