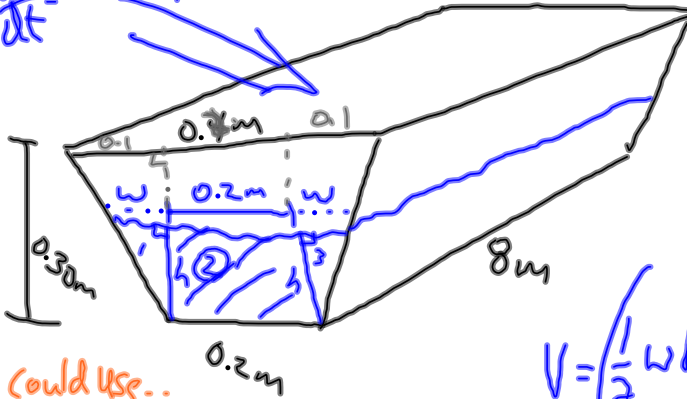The trough down the centre of a cattle barn is 40 cm wide at the top and 20 cm at the bottom. It is 30 cm deep and 8 m long. The trough is being filled at the rate of 0.25 m³/min. How fast is the water level in the trough rising when the water is 20 cm deep in the trough?

$\frac{dV}{dt} = 0.25 m^3/min.$

$V_{PRISM} = \dfrac{Area\ of}{Face} \times Length$

0.1 0.1m 0.1
w 0.2m w
0.30m
h
0.2m

Could use..

Area of a Trapezoid

$A = \frac{1}{2}(a+b)h$

a
h
b

Similar Shapes

Trough
0.1
0.3
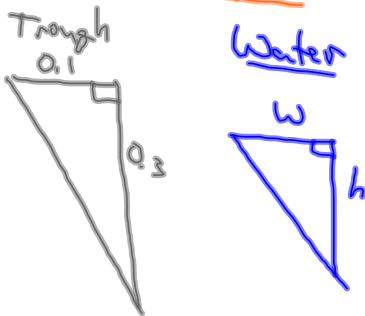
Water
w
h

$\frac{0.1}{w} = \frac{0.3}{h}$

$\frac{0.3w}{0.3} = \frac{0.1h}{0.3}$

$w = \frac{1}{3}h$

$V = \left(\frac{1}{2}wh + 0.2h + \frac{1}{2}wh\right)8$

$V = (wh + 0.2h)8$

$V = 8wh + 1.6h$ ← 3 variables

$V = 8\left(\frac{1}{3}h\right)h + 1.6h$

$V = \frac{8}{3}h^2 + 1.6h$

$\frac{dV}{dt} = \frac{16}{3}h\frac{dh}{dt} + 1.6\frac{dh}{dt}$

$0.25 = \frac{16}{3}(0.2)\frac{dh}{dt} + 1.6\frac{dh}{dt}$

$0.25 = 2.667\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{0.25}{2.667} = 0.09375\ m/min.$

1

Warm Up

A spherical hailstone is increasing in size at the rate of 2 cm^3/min. as it falls from the clouds. Determine the rate at which the radius of the hailstone is increasing at the instant it has a surface area of 4π cm^2. $SA = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$ Answer: $\frac{1}{2\pi}$ cm/min. [5]
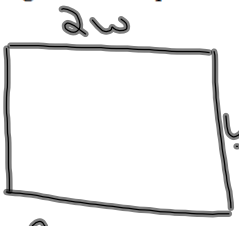
$V = \frac{4}{3}\pi r^3$

$4\pi = 4\pi r^2$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$1 = r^2$

$2 = 4\pi(1)\frac{dr}{dt}$ $\rightarrow \frac{dr}{dt} = \frac{2}{4\pi}$ cm/min $= \frac{1}{2\pi}$ cm/min.

A rectangle is expanding so that its length is always twice its width. The perimeter of the rectangle is increasing at a rate of 6 cm/min. Find the rate of increase of the area of the rectangle when the perimeter is 40 cm. [5]

$2w$ [rectangle with width w]

$P = 6w$

$\frac{dP}{dt} = 6\frac{dw}{dt}$

$6 = 6\frac{dw}{dt}$

$1 = \frac{dw}{dt}$
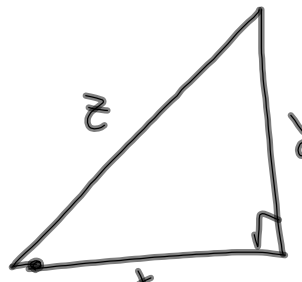
$A = 2w^2$

$\frac{dA}{dt} = 4w\frac{dw}{dt}$

$\frac{dA}{dt} = 4\left(\frac{20}{3}\right)(1) = \frac{80}{3}$ cm²/min.

$P = 6w$

$\frac{40}{6} = w$

A dog running at 5 m/s is chasing a squirrel running at the rate of 4 m/s. The squirrel dashes up a telephone pole in attempt to get away from the canine. If the dog is 16 m behind the squirrel when it begins to climb the pole, what is the rate at which the distance between the two animals is increasing 2 seconds after the squirrel starts up the pole? [5]

$\frac{1}{5}$ m/s

[triangle with legs x, y and hypotenuse z]

$x^2 + y^2 = z^2$

$\frac{dy}{dt} = 4$ m/s

$\frac{dy}{dt} = -5$ m/s

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$z = \sqrt{100}$
$z = 10m$

$8m = y$

$x = 6m$

$6(-5) + 8(4) = 10\frac{dz}{dt}$

$2 = 10\frac{dz}{dt}$

$\frac{1}{5}$ m/s $= \frac{dz}{dt}$

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#3 - 17 (omit 16)