Practice Problems...

A cubic block of ice (*which remains in the shape of a cube*) is melting so that its volume is *decreasing* at a rate of 2 cm^3/min. How fast is the length of a side changing (in cm/min) when the sides are 10 cm?

(a) $-\frac{2}{300}$ (b) None of these (c) $\frac{1}{600}$ (d) $\frac{2}{300}$ (e) $-\frac{1}{600}$

(Texas A & M Univ. Final Exam '12)

12. (10 pts) Water is poured into a conical cup at the rate of $\frac{5}{2}$ cubic inches per second. If the cup is 6 inches tall and the top of the cup has a radius of 2 inches, how fast does the water level rise when the water is 2 inches deep? Be sure to include units with your answer. NOTE: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

(Texas A & M Univ. Final Exam '08)

20. (7 pts) Cyclist A starts at point P and rides north at 15 mph. At the same time, cyclist B starts 10 miles ea of point P and rides east at 15 mph. How fast is the distance between them changing after 2 hours?

(Texas A & M Univ. Final Exam '12)

15. (10 points) Two honeybees sitting together in a hive start flying in search of flower juice. One flies north at 3 ft/s and the other flies east at 4 ft/s. At what rate is the distance between the honeybees increasing 10 seconds later.

(Kansas State University: Final Exam 08)

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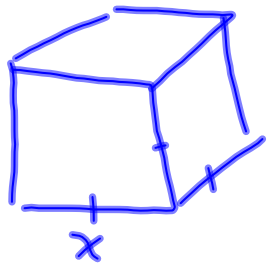

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$-2 = 3(10)^2 \frac{dx}{dt}$

$-\frac{2}{300}\ \text{cm/min} = \frac{dx}{dt}$

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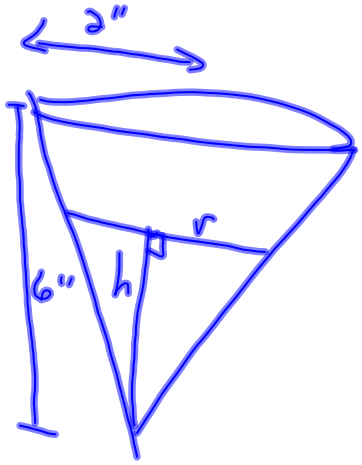
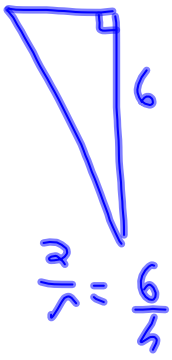
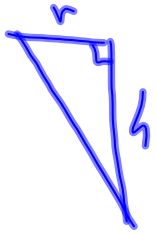


Cup



Water



$\frac{2}{r} = \frac{6}{h}$

$2h = 6r$

$\frac{2}{6}h = r$

$\frac{1}{3}h = r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

$V = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

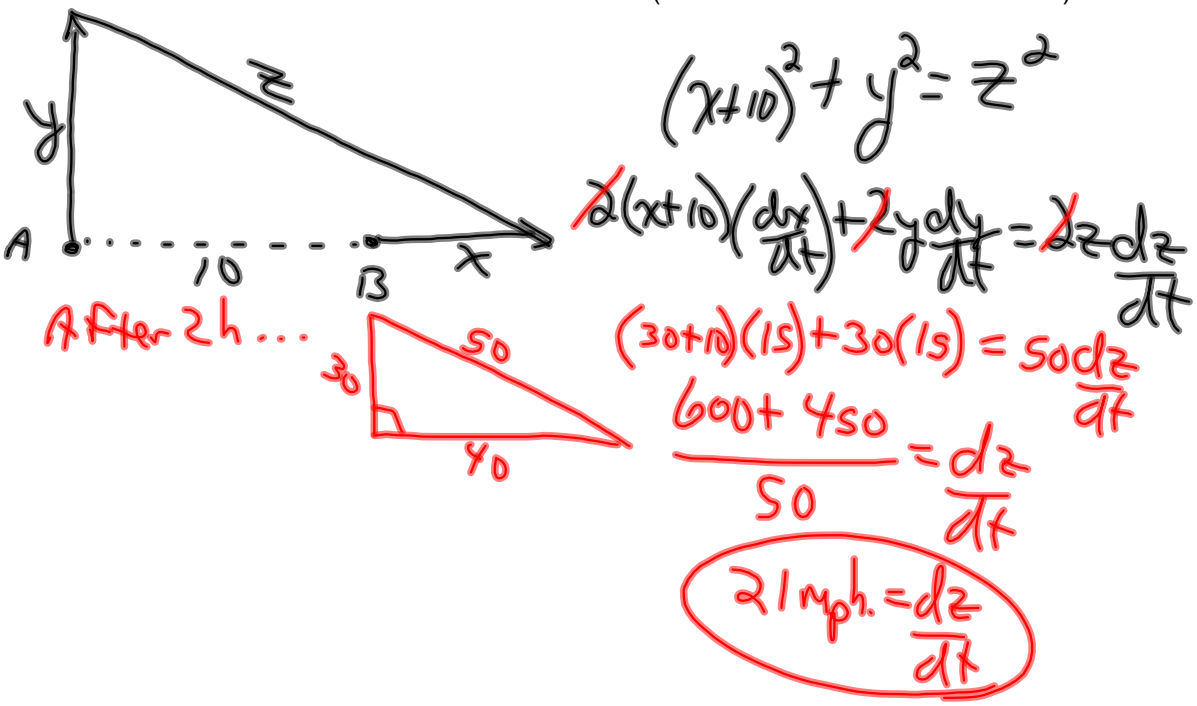
$\frac{5}{2} = \frac{1}{9}\pi(2)^2 \frac{dh}{dt}$

$\frac{45}{(8\pi)} = \frac{dh}{dt}$

$1.79\ \text{inches/sec} = \frac{dh}{dt}$

(7 pts) Cyclist A starts at point P and rides north at 15 mph. At the same time, cyclist B starts 10 miles east of point P and rides east at 15 mph. How fast is the distance between them changing after 2 hours?

(Texas A & M Univ. Final Exam '12)



$$(x+10)^2 + y^2 = z^2$$

$$2(x+10)\left(\frac{dx}{dt}\right) + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

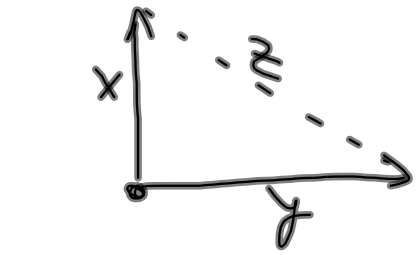
$$(30+10)(15) + 30(15) = 50\frac{dz}{dt}$$

$$\frac{600 + 450}{50} = \frac{dz}{dt}$$

$$21 \text{ mph} = \frac{dz}{dt}$$

15. (10 points) Two honeybees sitting together in a hive start flying in search of flower juice. One flies north at 3 ft/s and the other flies east at 4 ft/s. At what rate is the distance between the honeybees increasing 10 seconds later.

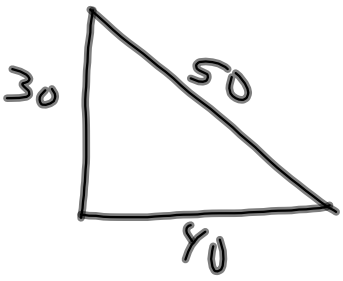
(Kansas State University: Final Exam 08)



$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

10 sec later...



$$30(3) + 40(4) = 50\frac{dz}{dt}$$

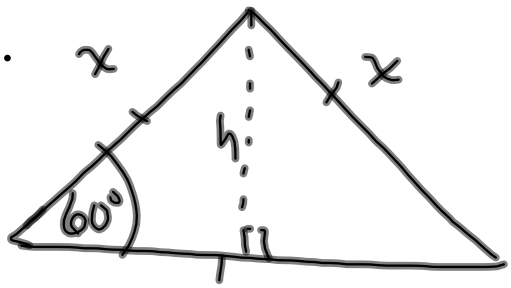
$$\frac{250}{50} = \frac{dz}{dt}$$

$$5 = \frac{dz}{dt}$$

$$5 \text{ ft./sec} = \frac{dz}{dt}$$

Textbook: Pgs. 145-146

8.



$\frac{dx}{dt} = -2\ cm/s$

$A = \frac{1}{2}bh$ OR $A = \frac{1}{2}ab\sin\theta$

$A = \frac{1}{2}x \cdot h \sin 60^\circ$

$A = \frac{1}{2}x^2 \sin 60^\circ$

$A = \frac{1}{2}x^2\left(\frac{\sqrt{3}}{2}\right)$

$A = \frac{\sqrt{3}}{4}x^2$

$\left(\frac{x}{2}\right)^2 + h^2 = x^2$

$h^2 = \frac{x^2}{1} - \frac{x^2}{4}$

$\sqrt{h^2} = \sqrt{\frac{3x^2}{4}}$

$h = \frac{\sqrt{3}x}{2}$

$A = \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right)$

$A = \frac{\sqrt{3}}{4}x^2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{2} \times \frac{dx}{dt}$

When $A = 100\ cm^2$

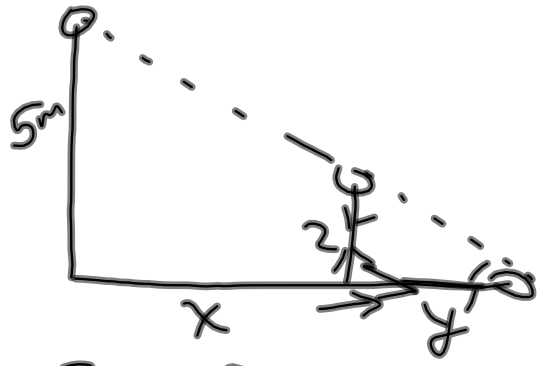
$100 = \frac{\sqrt{3}}{4}x^2$

$\sqrt{\frac{400}{\sqrt{3}}} = x$

$\frac{dA}{dt} = \frac{\sqrt{3}}{2}\left(\sqrt{\frac{400}{\sqrt{3}}}\right)(-2)$

$= \frac{-\sqrt{3}(20)}{\sqrt{\sqrt{3}}} = -26.3\ cm^2/sec$

10.



$$\frac{5}{x+y} = \frac{2}{y}$$

$$5y = 2x + 2y$$

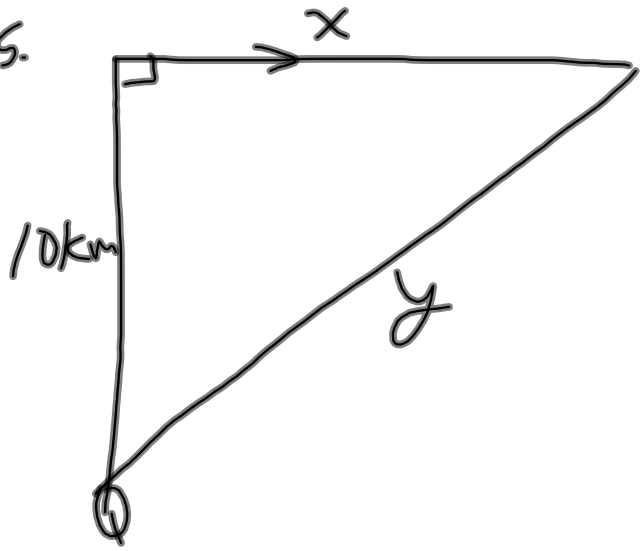
$$3y = 2x$$

$$3\frac{dy}{dt} = 2\frac{dx}{dt}$$

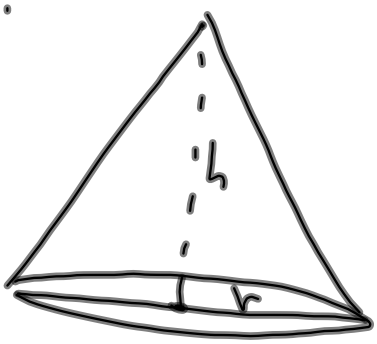
$$3(1.5) = 2\frac{dx}{dt}$$

$$2.25\,m/s = \frac{dx}{dt}$$

15.



17.



$d = h$

$2r = h$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$1.2 = \frac{1}{4}\pi(3)^2 \frac{dh}{dt}$

$\frac{4.8}{(9\pi)} = \frac{dh}{dt}$

$\frac{dh}{dt} = 0.17 \text{ m/min.}$