Absolute Maxima/Minima

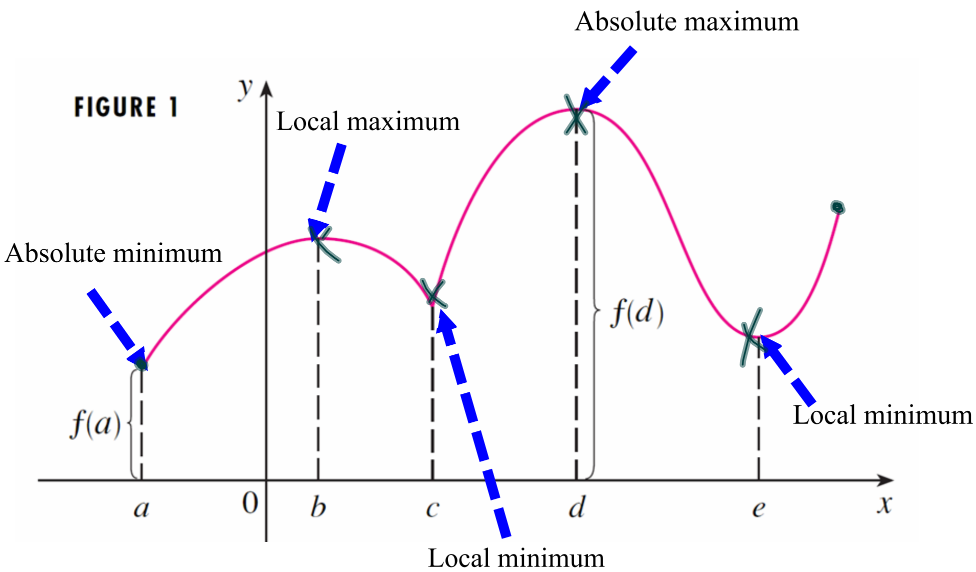
A function f has an absolute (or global) maximum at c if $f(c) \geq f(x)$ for all x in the domain D of f.

- The number $f(c)$ is called the maximum value of f on D.

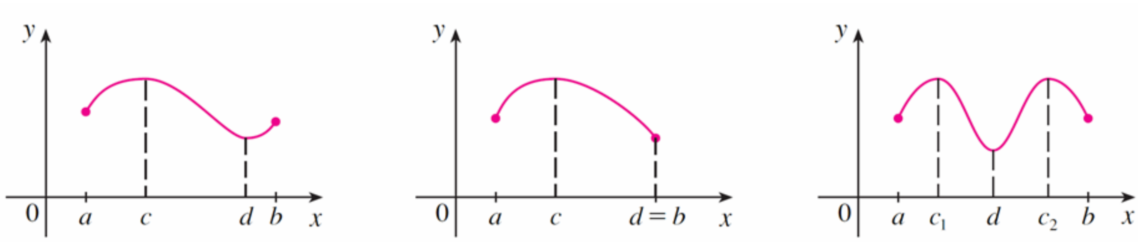
A function f has an absolute (or global) minimum at c if $f(c) \leq f(x)$ for all x in the domain D of f.

- The number $f(c)$ is called the minimum value of f on D.

FIGURE 1



$\boxed{3}$ The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.



Here are a couple of examples to reinforce that the function must be continuous over a closed interval.

Discontinuous at $x = 1$.

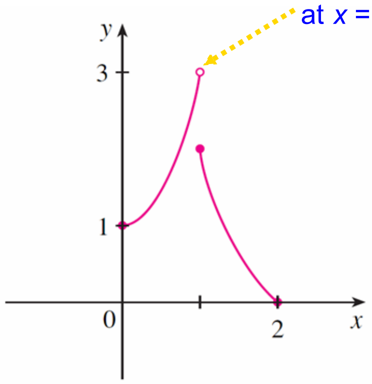


FIGURE 6
This function has minimum value $f(2) = 0$, but no maximum value.

Interval is open at both ends

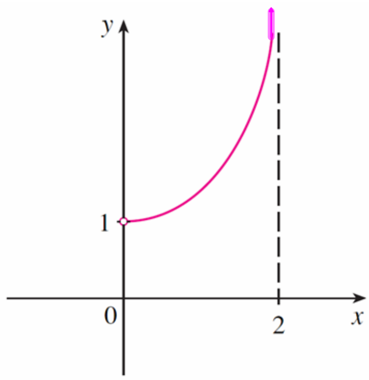


FIGURE 7
This continuous function g has no maximum or minimum.

How do we find extreme values?

4 **Fermat's Theorem** If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

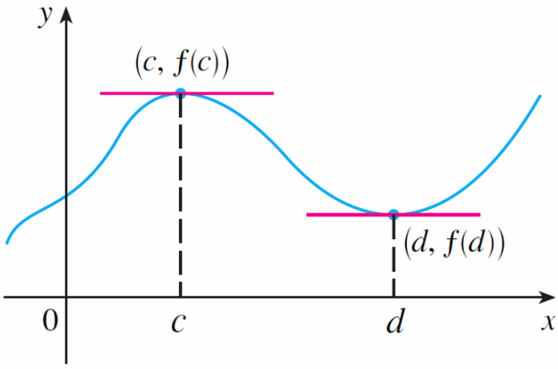


FIGURE 8

The converse of the theorem is false in general:

- Look at $f(x) = x^3$

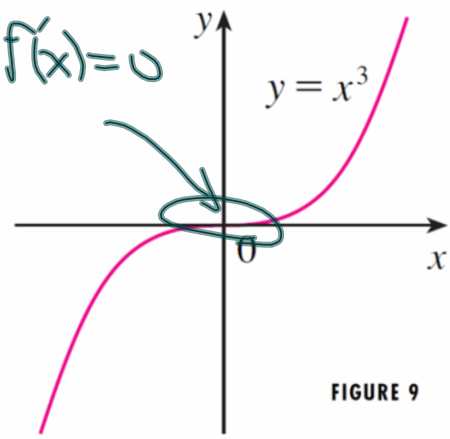


FIGURE 9

There can also be an extreme value wher $f'(c)$ does not exist.

Look at the function $f(x)=|x|$

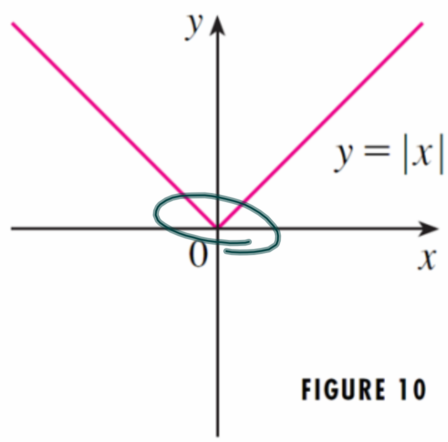


FIGURE 10

5 Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Example:

Find the critical values of $f(x) = x^{\frac{3}{5}}(4 - x)$ and determine all intervals of increase and decrease as well as any local extrema.

$$f'(x) = \frac{3}{5}x^{-2/5}(4-x) + x^{3/5}(-1)$$

$$f'(x) = \frac{3}{5}x^{-2/5}(4-x) - x^{3/5}$$

$$f'(x) = x^{-2/5}\left[\frac{3}{5}(4-x) - x\right]$$

$$\frac{x^{3/5}}{x^{-2/5}}$$

$x^2 - x^5$

$x^2(1 - x^3)$

lowest

$$f'(x) = \frac{1}{x^{2/5}}\left(\frac{12}{5} - \frac{3}{5}x - x\right)$$

$$f'(x) = \frac{1}{x^{2/5}}\left(\frac{12}{5} - \frac{8}{5}x\right)$$

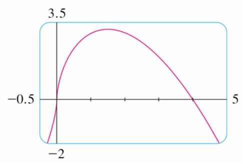


FIGURE 11

Critical Values

$x = 0$ or $\frac{12}{5} - \frac{8}{5}x = 0$

$$12 - 8x = 0$$

$$\frac{12}{8} = \frac{8x}{8}$$

$$\frac{3}{2} = x$$

	$x^{2/5}$	$\frac{12}{5} - \frac{8}{5}x$	f'	f
$(-\infty, 0)$	+	+	+	Inc
$(0, \frac{3}{2})$	+	+	+	Inc
$(\frac{3}{2}, \infty)$	+	−	−	Dec

Local Max

Local Max: $\left(\frac{3}{2}, f\left(\frac{3}{2}\right)\right)$ Local Min. (None)

How do we determine absolute maximum and minmum values?

The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Example:

Given the function...

$$f(x) = 3x^4 - 16x^3 + 18x^2 \ , \ -1 \le x \le 4$$

Determine the absolute maximum and minimum values of the function.

Solution

$$f'(x) = 12x^3 - 48x^2 + 36x$$

$$0 = 12x(x^2 - 4x + 3)$$

$$0 = 12x(x-3)(x-1)$$

$x = 0, 3, 1 \longrightarrow$ Closed Interval $[-1, 4]$

x	y
-1	
0	
1	
3	
4	