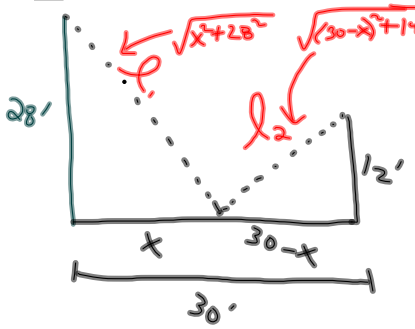
Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached at a single stake, running from ground level to the top of each post. Where should the stake be placed to use the least amount of wire?



distance $= \ell_1 + \ell_2$

$$\ell = \sqrt{x^2+28^2} + \sqrt{900-60x+x^2+144}$$

$$\ell' = \frac{1}{2}(x^2+28^2)^{-1/2}(2x) + \frac{1}{2}(x^2-60x+1044)^{-1/2}(2x-60)$$

$$0 = \frac{x}{\sqrt{x^2+784}} + \frac{x-30}{\sqrt{x^2-60x+1044}}$$

$$\left(\frac{-x}{\sqrt{x^2+784}}\right)^2 = \left(\frac{x-30}{\sqrt{x^2-60x+1044}}\right)^2$$

$$\frac{x^2}{x^2+784} = \frac{(x-30)^2}{x^2-60x+1044}$$

$$x^4-60x^3+1044x^2 = (x^2+784)(x^2-60x+900)$$

$$x^4-60x^3+1044x^2 = x^4-60x^3+900x^2+784x^2-47040x+705600$$

$$0 = 640x^2-47040x+705600$$

$$0 = 64x^2-4704x+70560$$

$$x = \frac{4704 \pm \sqrt{(4704)^2-4(64)(70560)}}{2(64)}$$

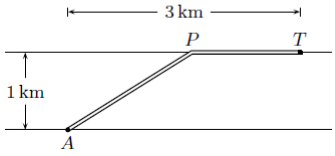
$$x = \frac{4704 \pm 2016}{128}$$

$x = 525$ or 21 (525: too large)

$\Rightarrow$ 21' from 28' pole

4) 13. Find the absolute extrema of $f(x) = (2x-1)\sqrt[3]{x}$ on $[-1, 1]$.

5) 14. An oil company has a refinery at point A on the bank of a straight river 1 kilometer wide. It is going to run a pipe from point A to point P somewhere on the opposite side of the river, and then straight along the river to a tank T situated 3 kilometers downstream from A. It costs 15 thousand dollars per kilometer to run the pipe under the water and 9 thousand dollars per kilometer to run the pipe along the bank. What should be the distance from P to T in order to minimize the total cost of the pipe?



13. $f'(x) = 2x^{1/3} + (2x-1)\left(\frac{1}{3}x^{-2/3}\right)$

$0 = x^{-2/3}\left[2x + \frac{1}{3}(2x-1)\right]$

$0 = x^{-2/3}\left(2x + \frac{2}{3}x - \frac{1}{3}\right)$

$\frac{1}{x^{2/3}} = 0$

$\frac{8}{3}x - \frac{1}{3} = 0$

$x = 0$

$8x - 1 = 0$

$8x = 1$

$x = \frac{1}{8}$

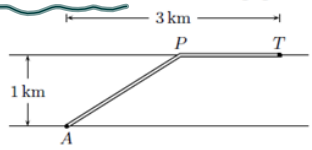
$[-1, 1]$

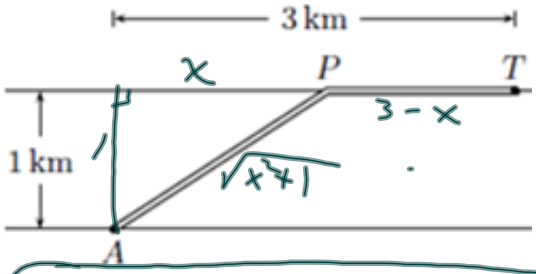
x	y
-1	3
0	0
$\frac{1}{8}$	$-\frac{3}{8}$
1	1

Abs. Max. = 3

Abs. Min. = $-\frac{3}{8}$

An oil company has a refinery at point A on the bank of a straight river 1 kilometer wide. It is going to run a pipe from point A to point P somewhere on the opposite side of the river, and then straight along the river to a tank T situated 3 kilometers downstream from A. It costs 15 thousand dollars per kilometer to run the pipe under the water and 9 thousand dollars per kilometer to run the pipe along the bank. What should be the distance from P to T in order to minimize the total cost of the pipe?





Cost = $15000\sqrt{x^2+1} + 9000(3-x)$

$C = 15000(x^2+1)^{1/2} + 27000 - 9000x$

$C' = 7500(x^2+1)^{-1/2}(2x) - 9000$

$0 = \frac{15000x}{\sqrt{x^2+1}} - 9000$

$9000 = \frac{15000x}{\sqrt{x^2+1}}$

$\frac{(9000)^2}{1} = \frac{15000^2 x^2}{x^2+1}$

$9000^2 x^2 + 9000^2 = 15000^2 x^2$

$9000^2 = 144000000 x^2$

$\sqrt{0.5625} = \sqrt{x^2}$

$0.75 = x$

$\overline{PT} = 2.25$ km

Practice Problems:

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#1, 2, 3, 5, 6, 7, 8, 10, 11
12, 16, 17, 20