

Practice...

Factor each of the following:

1. $10x^2y^5 + 20x^7y^3 - 25x^4y^9$

$$5x^2y^3(2y^2 + 4x^5 - 5x^3y^6)$$

3. $x^2 - 10x + 24$

$$(x-6)(x-4)$$

5. $3m^2 - 24m - 27$

$$3(m^2 - 8m - 9)$$
$$3(m-9)(m+1)$$

2. $2x^2 + 15y - 5x - 6xy$

"Grouping"

$$2x^2 - 5x + 15y - 6xy$$
$$x(2x-5) - 3y(-5+2x)$$
$$(2x-5)(x-3y)$$

4. $m^2 + 13m - 30$

$$(m+15)(m-2)$$

6. $x^2(a+3) + 2x(a+3) - 48(a+3)$

$$(a+3)(x^2 + 2x - 48)$$
$$(a+3)(x+8)(x-6)$$

Warm Up

Factor each of the following:

$$25a^2 - 9b^2$$

$$(5a - 3b)(5a + 3b)$$

$$81w^{16} - 1$$

$$(9w^8 - 1)(9w^8 + 1)$$

$$(3w^4 - 1)(3w^4 + 1)(9w^8 + 1)$$

$$\sqrt{(2x-1)^2 - 16}$$

$$4x^2 - 4x + 1 - 16$$

$$4x^2 - 4x - 15$$

$$4x^2 - 10x + 6x - 15$$

$$2x(2x-5) + 3(2x-5)$$

$$(2x-5)(2x+3)$$

$$[(2x-1)-4][(2x-1)+4]$$

$$(2x-5)(2x+3)$$

$$100x^2 - 140xy + 49y^2$$

$$(10x - 7y)^2$$

Other methods of factoring...

- Combinations of techniques

EXAMPLES....

$$\begin{array}{l}
 1) \quad \underbrace{x^2 - 10x + 25 - y^2} \\
 \quad \quad (x-5)^2 - y^2 \\
 \quad \quad [(x-5) - y][(x-5) + y] \\
 \quad \quad (x-y-5)(x+y-5)
 \end{array}
 \quad
 \begin{array}{l}
 2) \quad \underbrace{25x^2 - 8n - n^2 - 16} \\
 \quad \quad 25x^2 - (n^2 + 8n + 16) \\
 \quad \quad 25x^2 - (n+4)^2 \\
 \quad \quad [5x - (n+4)][5x + (n+4)] \\
 \quad \quad (5x - n - 4)(5x + n + 4)
 \end{array}$$

Recall Grouping: $\underbrace{4ax^2 - 9a} - \underbrace{4x^2y + 9y}$

$$\begin{array}{l}
 a(4x^2 - 9) - y(4x^2 - 9) \\
 (4x^2 - 9)(a - y) \\
 \boxed{(2x-3)(2x+3)(a-y)}
 \end{array}$$

Give these a try...

Examples:

$$(a) \quad a^2 + 2ab + b^2 - 9$$
$$(a+b)^2 - 9$$
$$(a+b-3)(a+b+3)$$

$$(c) \quad x^2 - 14x + 49 - y^2$$
$$(x-7)^2 - y^2$$
$$(x-7-y)(x-7+y)$$

$$(b) \quad 9 - (a^2 + 2ab + b^2)$$
$$9 - (a+b)^2$$
$$(3-(a+b))(3+(a+b))$$
$$(3-a-b)(3+a+b)$$

$$(d) \quad 4x^2 + 20x + 25 - 9y^2$$
$$(2x+5)^2 - 9y^2$$
$$(2x+5-3y)(2x+5+3y)$$

$$\begin{aligned}
 &1. \quad \underline{2x^3 + 3x^2} - \underline{8x - 12} \\
 &x^2(2x+3) - 4(2x+3) \\
 &(x^2-4)(2x+3) \\
 &(x-2)(x+2)(2x+3)
 \end{aligned}$$

$$\begin{aligned}
 &2. \quad 3x^3 - 2x^2 - 3x + 2 \\
 &x^2(\underline{3x-2}) - 1(\underline{3x-2}) \\
 &(3x-2)(x^2-1) \\
 &(3x-2)(x-1)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 &\text{Factor } \underline{x^2 - 6x + 9} - y^2 \\
 &(x-3)^2 - y^2 \\
 &(x-3-y)(x-3+y)
 \end{aligned}$$

$$\begin{aligned}
 &a^3 - 3a^2 + 9a - 27 \\
 &a^2(a-3) + 9(a-3) \\
 &(a-3)(a^2+9)
 \end{aligned}$$

● Substitutions

*introduce a new variable to replace the parts that repeat in the polynomial.

- use a "Let" statement.
- factor the polynomial with the new variable.
- resubstitute into factored form.

EXAMPLES....

$$4m^2 - 18m + 8 \Rightarrow \text{Let } m = 2x - 3$$

1) $4(2x - 3)^2 - 18(2x - 3) + 8$

$$4m^2 - 18m + 8$$

$$2(2m^2 - 9m + 4)$$

$$2(2m^2 - 8m - 1m + 4)$$

$$2(2m(m-4) - 1(m-4))$$

$$2(m-4)(2m-1)$$

Resubstitute !!

$$2(2x-3-4)(2(2x-3)-1)$$

$$2(2x-7)(4x-7)$$

2) $a^4b^4 + 6a^2b^2 - 40$

$$(a^2b^2)^2 + 6(a^2b^2) - 40$$

$$\text{Let } m = a^2b^2$$

$$m^2 + 6m - 40$$

$$(m+10)(m-4)$$

$$(a^2b^2+10)(a^2b^2-4)$$

$$(a^2b^2+10)(ab-2)(ab+2)$$