Example 5 Factoring Trigonometric Expressions

Factor (a) $\sec^2\theta - 1$ and (b) $4\tan^2\theta + \tan\theta - 3$.

$\sin x \cos^2 x - \sin x$

$\tan^4 x + 2\tan^2 x + 1$

$\cot^2 x - \cot x - 2$

$\dfrac{\cos^2 x - 4}{\cos x - 2}$

$\sin^4 x - \cos^4 x$

$\sec^2 x \tan^2 x + \sec^2 x$

(a) $\sec^2\theta - 1$

$(\sec\theta)^2 - 1$

$(\sec\theta - 1)(\sec\theta + 1)$

$4\tan^2\theta + \tan\theta - 3$

$4(\tan\theta)^2 + \tan\theta - 3$ $\quad -12$

$4m^2 + m - 3$

$4\tan^2\theta + 4\tan\theta - 3\tan\theta - 3$

$4\tan\theta(\tan\theta + 1) - 3(\tan\theta + 1)$

$(\tan\theta + 1)(4\tan\theta - 3)$

$\sin x \cos^2 x - \sin x$

$\sin x(\cos^2 x - 1)$

$\sin(\cos x - 1)(\cos x + 1)$

$\dfrac{\tan^2 x}{\tan^4 x + 2\tan^2 x + 1}$ $\quad 1$

$(\tan^2 x + 1)^2$

$\cot^2 x - \cot x - 2$

$(\cot x - 2)(\cot x + 1)$

$\dfrac{\cos^2 x - 4}{\cos x - 2}$

$\dfrac{(\cos x - 2)(\cos x + 2)}{\cos x - 2}$

$= \cos x + 2$

$\sin^4 x - \cos^4 x$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$

$(\sin x - \cos x)(\sin x + \cos x)(\sin^2 x + \cos^2 x)$

$\sec^2 x \tan^2 x + \sec^2 x$

$\sec^2 x (\tan^2 x + 1)$

1. p) $3^{2x} + 5(3^x) + 6$

$(3^x)^2 + 5(3^x)^1 + 6$

Let $m = 3^x$

$m^2 + 5m + 6$

$(m+3)(m+2)$

$(3^x+3)(3^x+2)$

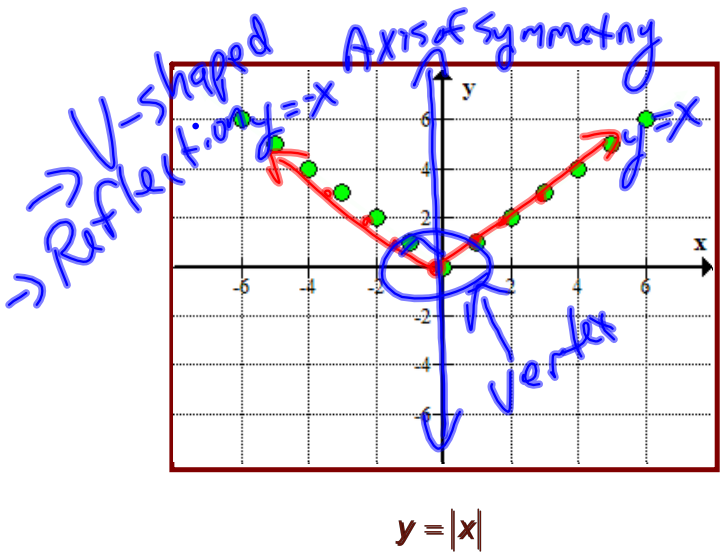
Chapter 7

Graph an Absolute Value Function $y = |f(x)|$

Use the pen tool to complete the table of values for the function $y = |x|$ and draw the corresponding graph.

$y = |x|$

x	y
–3	3
–2	2
–1	1
0	0
1	1
2	2
3	3



$y = |x|$

Hint

Discuss with a partner the key features of the graph of $y = |x|$. To reveal the answer, click the function below the graph.

Chapter 7

Graphing an Absolute Value Function

Drag the red points and line to the grid to sketch the graph of $y = 3x + 4$. Then, drag the blue points and ray to the grid to sketch the graph of $y = |3x + 4|$.

x	y
0	4
2	10
-1	1

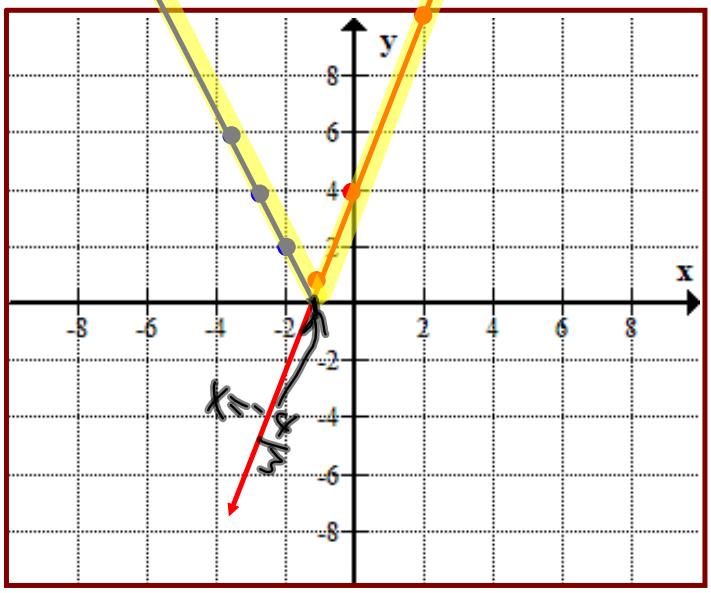
$y = 3x + 4$

$0 = 3x + y$

$-y = 3x$

$-\frac{y}{3} = x$

$y = |3x + 4|$

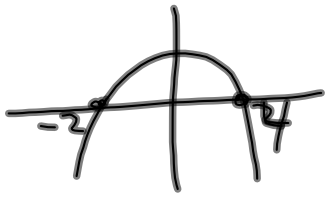


Hint

Click here for the solution.

Sketching a quadratic function...

- Use vertex and zeros



Examples:

$y = x^2 - x - 6$

$y = x(x-1) - 6$

$x = 0 \qquad x = 1$

vertex $x \Rightarrow \frac{0+1}{2}$

$x \Rightarrow \frac{1}{2}$

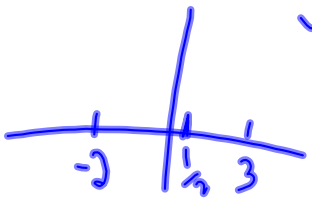
y vertex $\Rightarrow \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 6$

OR

$0 = x^2 - x - 6$

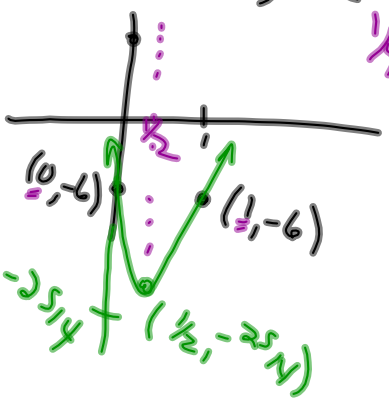
$0 = (x-3)(x+2)$

$x = 3, -2$



$y = \frac{1}{4} - \frac{1}{2} - 6 = -\frac{25}{4}$

$\left(\frac{1}{2}, -\frac{25}{4}\right)$

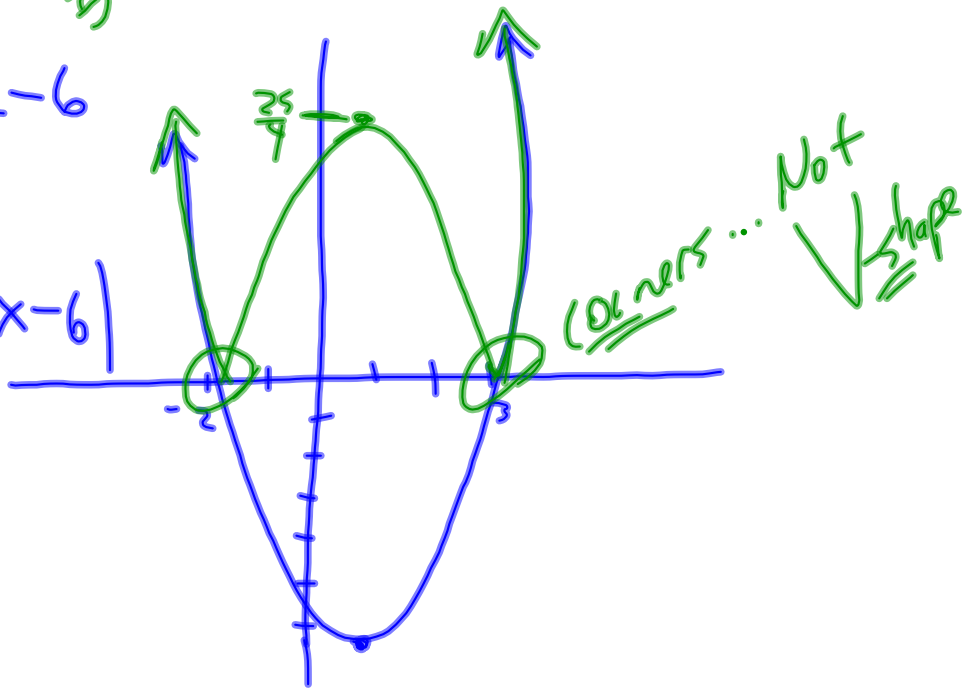


$\frac{1}{4} - \frac{1}{2} - 6 = -\frac{25}{4}$

$y = x^2 - x - 6$

Sketch:

$y = |x^2 - x - 6|$



$$y = -x^2 + 5x + 14$$

Sketch ↑ then $y = |-x^2 + 5x + 14|$

Partial Factoring:

$y = -x(x-5) + 14$

$x = 0 \quad x = 5$