Differentiate:

$$y = \frac{3x^6\sqrt{x^5 - \sqrt[3]{3x+2}}}{\sqrt[7]{1-(8x^3)(x^7+3)^8}}$$

$$y' = \frac{\left[18x^5\sqrt{x^5-\sqrt[3]{3x+2}} + 3x^6\left[\frac{1}{2}\left(x^5-\sqrt[3]{3x+2}\right)^{-1/2}\left(5x^4-\frac{1}{3}(3x+2)^{-2/3}(3)\right)\right]\right]\sqrt[7]{1-(8x^3)(x^7+3)^8} - \left[3x^6\sqrt{x^5-\sqrt[3]{3x+2}}\right]\left[\frac{1}{7}\left(1-(8x^3)(x^7+3)^8\right)^{-6/7}\cdot\left((-24x^2)(x^7+3)^8+(-8x^3)\left[8(x^7+3)^7(7x^6)\right]\right)\right]}{\left(\sqrt[7]{1-(8x^3)(x^7+3)^8}\right)^2}$$

$$y = \frac{8}{3x}$$

$$y = 8(3x)^{-1}$$

$$y' = -8(3x)^{-2}(3)$$

$$y' = \frac{-8\ (3)}{(3x)^2}$$

$$y' = -\frac{8(\cancel{3})}{\underset{3}{\cancel{9}}x^2}$$

$$y' = -\frac{8}{3x^2}$$

$$y' = -\frac{8}{3}x^{-2}$$

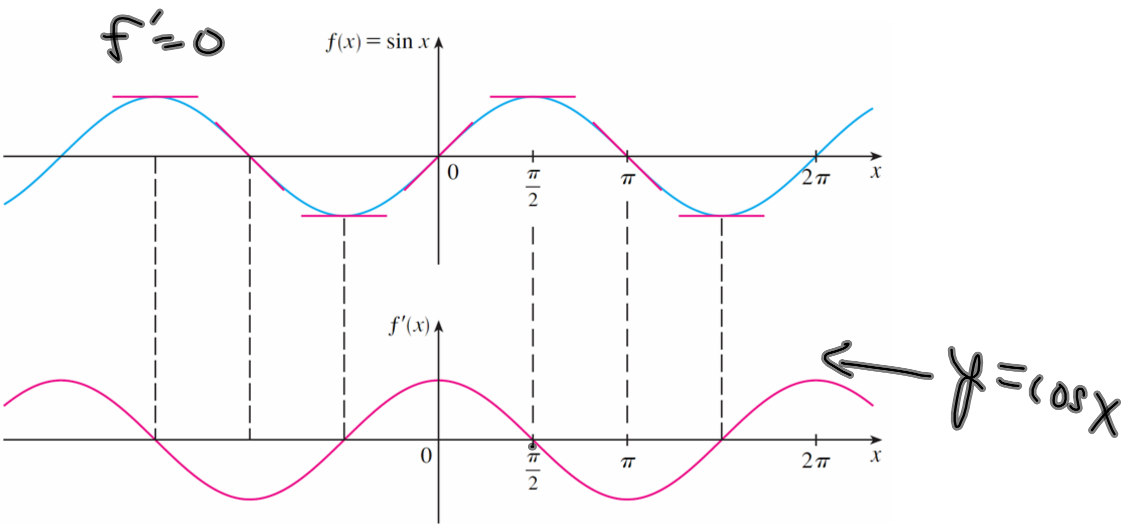
$$y = \frac{8}{3x}$$

$$y = \frac{8}{3}x^{-1}$$

$$y' = -\frac{8}{3}x^{-2}$$

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



$y = \sin x$

Let's check this using the definition of a derivative...

$f(x+h) = \sin(x+h)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

"0"

1

$$= \lim_{h \to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h \to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h \to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h \to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h \to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du \qquad \frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du \qquad \frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du \qquad \frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

ex. $y = \sin \underbrace{(3x^7)}_{u}$

$y' = \cos(3x^7)(21x^6)$

$y = \sec \sqrt{x}$

$y' = \sec\sqrt{x} \tan\sqrt{x} \left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$y = \cos^3 5x$

$y = (\cos 5x)^3$

$y' = 3(\cos 5x)^2\left(-\sin(5x)(5)\right)$

$y = \csc^4 x^{-3} \rightarrow (\csc x^{-3})^4$

$y' = 4\csc^3 x^{-3}\left(-\csc x^{-3} \cot x^{-3}(-3x^{-4})\right)$

$\downarrow$

$(\csc x^{-3})^3$

Differentiate each of the following:

1. $f(x) = \cos\underbrace{\sqrt{5x-1}}_{u} + \tan x^3$

$f'(x) = -\sin\sqrt{5x-1}\left[\frac{1}{2}(5x-1)^{-1/2}(5)\right] + \sec^2 x^3(3x^2)$

2. $y = \dfrac{\sec(5x)}{\cot\sqrt{x}} = \sec 5x \tan\sqrt{x}$

$y' = \dfrac{\left[\sec 5x \tan 5x\,(5)\right]\cot\sqrt{x} - \sec 5x\left(-\csc^2\sqrt{x}\left(\frac{1}{2}x^{-1/2}\right)\right)}{\cot^2\sqrt{x}}$

3. $f(x) = \csc^2\sqrt{x} - \sqrt{\sin(9x^6)}$

$f'(x) = 2(\csc\sqrt{x})^1\left(-\csc\sqrt{x}\cot\sqrt{x}\left(\frac{1}{2}x^{-1/2}\right)\right) - \frac{1}{2}(\sin 9x^6)^{-1/2} \cdot \left[\cos 9x^6\,(54x^5)\right]$

4. $f(x) = \tan\underbrace{\left[\cos\left(8x^{-3}\right)\right]}_{u}$

$f'(x) = \sec^2\left[\cos(8x^{-3})\right]\left[-\sin(8x^{-3})(-24x^{-4})\right]$

5. $f(x) = \sin\underbrace{\left\{\cos\left[\tan^3(7x)\right]\right\}}_{u}$

$f'(x) = \cos\left\{\cos\left[\tan^3(7x)\right]\right\}\left[-\sin(\tan^3 7x)\left[3(\tan 7x)^2 \cdot \sec^2 7x(7)\right]\right]$

6. $y = \dfrac{6x^3\sqrt{5\cot\sqrt{x} + \cos^3 3x}}{\tan\left(\sin\sqrt[3]{x}\right) + 8\cot x^7 - \csc\left(x^4 - 1\right)^5}$

$$y' = \frac{\left[18x^2\sqrt{5\cot\sqrt{x}+\cos^3 3x} + 6x^3\left[\frac{1}{2}\left(5\cot\sqrt{x}+\cos^3 3x\right)^{-\frac{1}{2}}\left(-5\csc^2\sqrt{x}\left(\frac{1}{2}x^{-\frac{1}{2}}\right) + 3(\cos 3x)^2(-\sin 3x(3))\right)\right]\right]\left[\tan(\sin\sqrt[3]{x}) + 8\cot x^7 - \csc(x^4-1)^5\right] - \left[6x^3\sqrt{5\cot\sqrt{x}+\cos^3 3x}\right]\left[\sec^2(\sin\sqrt[3]{x})\left(\cos\sqrt[3]{x}\left(\frac{1}{3}x^{-\frac{2}{3}}\right)\right) + -8\csc^2 x^7(7x^6) + \csc(x^4-1)^5\cot(x^4-1)^5\left(5(x^4-1)^4(4x^3)\right)\right]}{\left[\tan(\sin\sqrt[3]{x}) + 8\cot x^7 \csc(x^4-1)^5\right]^2}$$