Warm Up ... <u>Differentiate:</u>

$$f(x) = \frac{\sqrt{\sin 5x - \sec x^3}}{\cos^3(\sqrt{x}) + \tan(\csc\sqrt{x})\cot(x^{-6})}$$

$$f'(x) = \frac{\left[\frac{1}{2}\left(\sin 5x - \sec x^3\right)^{-1/2}\left(\cos 5x(5) - \sec x^3 \tan x^3 (3x^2)\right)\right] \cdot \left[\cos^3\sqrt{x} + \tan(\csc\sqrt{x})\cot x^{-6}\right] - \left[\sqrt{\sin 5x - \sec x^3}\right] \cdot \left[3\cos^2\sqrt{x}(-\sin\sqrt{x})\left(\tfrac{1}{2}x^{-1/2}\right) + \sec^2(\csc\sqrt{x})\left(-\csc\sqrt{x}\cot\sqrt{x}\left(\tfrac{1}{2}x^{-1/2}\right)\right)\cot x^{-6} + \tan(\csc\sqrt{x})\left(-\csc^2 x^{-6}(-6x^{-7})\right)\right]}{\left(\cos^3\sqrt{x} + \tan(\csc\sqrt{x})\cot x^{-6}\right)^2}$$

$(\cos\sqrt{x})^2$

Example

- For the folium of Descartes $x^3 + y^3 = 6xy$,
  - Find y'
  - Find the tangent to the curve at the point (3, 3)
  - At what points on the curve is the tangent line horizontal?

$(y)^3$

$3(y)^2\left(\frac{dy}{dx}\right)$

$$3x^2 + 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$$

$$\frac{dy}{dx}(3y^2 - 6x) = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$$

2) @(3,3) slope??

$$@(3,3)\ \frac{dy}{dx} = \frac{2(3) - (3)^2}{3^2 - 2(3)} = -1$$

$$y - 3 = -1(x - 3)$$

$$y - 3 = -x + 3$$

$$y = -x + 6$$

3) Horizontal Tangent??

$$\frac{dy}{dx} = 0$$

② $\frac{2y - x^2}{y^2 - 2x} = 0$

$$2y - x^2 = 0$$

$$y = \frac{x^2}{2}$$

① $x^3 + y^3 = 6xy$

$$x^3 + \left(\frac{x^2}{2}\right)^3 = 6x\left(\frac{x^2}{2}\right)$$

$$x^3 + \frac{x^6}{8} = 3x^3$$

$$x^3 - 3x^3 + \frac{x^6}{8} = 0$$

$$8x^3 - 24x^3 + x^6 = 0$$

$$-16x^3 + x^6 = 0$$

$$x^3(-16 + x^3) = 0$$

$x^3 = 0$

$x = 0$

$y = \frac{(0)^2}{2}$

$(0,0)$

or $-16 + x^3 = 0$

$$x^3 = 16$$

$$x = \sqrt[3]{16}$$

$$y = \frac{(\sqrt[3]{16})^2}{2}$$

$$\left(\sqrt[3]{16}, \frac{(\sqrt[3]{16})^2}{2}\right)$$

Try these... Find dy/dx

$$3x^2 - 6x^3y^4 = y^7 - 8x^3$$

$$6x - \left(18x^2y^4 + 6x^3(4y^3)\frac{dy}{dx}\right) = 7y^6\frac{dy}{dx} - 24x^2$$

$$\boxed{\frac{6x - 18x^2y^4 + 24x^2}{7y^6 + 24x^3y^3}} = \frac{\cancel{(7y^6 + 24x^3y^3)}\frac{dy}{dx}}{\cancel{(7y^6 + 24x^3y^3)}}$$

Example:

Given $x^2 - 3x^3y^2 + y^2 = 5xy^3 - 4$

Find $\frac{dy}{dx}$

$$2x - \left(9x^2y^2 + 3x^3\left(2y\frac{dy}{dx}\right)\right) + 2y\frac{dy}{dx} = 5y^3 + 5x(3y^2)\frac{dy}{dx}$$

$$2x - 9x^2y^2 - 5y^3 = \left(15xy^2 + 6x^3y - 2y\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x - 9x^2y^2 - 5y^3}{15xy^2 + 6x^3y - 2y}$$

Example:

Find $\frac{dy}{dx}$, given the curve $x^2 - 3xy = (5x^2 - 8y)^5$

$$2x - \left(3y + 3x\frac{dy}{dx}\right) = 5(5x^2-8y)^4\left(10x - 8\frac{dy}{dx}\right)$$

$$2x - 3y - 3x\frac{dy}{dx} = 50x(5x^2-8y)^4 - 40(5x^2-8y)^4\frac{dy}{dx}$$

$$\left[40(5x^2-8y)^4 - 3x\right]\frac{dy}{dx} = 50x(5x^2-8y)^4 - 2x + 3y$$

$$\frac{dy}{dx} = \frac{50x(5x^2-8y)^4 - 2x + 3y}{40(5x^2-8y)^4 - 3x}$$

$$\sqrt{x^3+7y^6} + 8x^3y = 4 + 3x^3y^5$$

$$\frac{1}{2}\left(x^3+7y^6\right)^{-1/2}\left(3x^2+42y^5\frac{dy}{dx}\right) + 24x^2y + 8x^3\frac{dy}{dx} = 9x^2y^5 + 3x^3\left(5y^4\frac{dy}{dx}\right)$$

$$\frac{3}{2}\left(x^3+7y^6\right)^{-1/2} + 21y^5\left(x^3+7y^6\right)^{-1/2} + 24x^2y + 8x^3\frac{dy}{dx} = 9x^2y^5 + 15x^3y^4\frac{dy}{dx}$$

$$\frac{dy}{dx} =$$

Homework

Page 107

1 d, f, h

2 c, d

3 c, d

5 a

6 a, b, c