Find the 3^{rd} derivative:

$$f(x) = \frac{3}{\sqrt{5-x^2}} = 3(5-x^2)^{-1/2}$$

$$f'(x) = -\frac{3}{\cancel{2}}(5-x^2)^{-3/2}(-\cancel{2}x) = 3(5-x^2)^{-3/2}(x)$$

$$f''(x) = \left[+\frac{9}{\cancel{2}}(5-x^2)^{-5/2}(\cancel{-2}x)\right](x) + 3(5-x^2)^{-3/2}$$

$$x^2$$

$$f'''(x) = \left[\left[-\frac{45}{2}(5-x^2)^{-7/2}(-2x)\right]x^2 + 9(5-x^2)^{-5/2}(2x)\right] + \ldots$$

$$-\frac{9}{2}(5-x^2)^{-5/2}(-2x)$$

Rectilinear Motion and Derivatives

Any motion along a straight line is called rectilinear motion.

Displacement - Velocity - Acceleration

"t" → time

If s represents a function that measures displacement, then $\frac{ds}{dt}$ would represent ???

$$velocity = \frac{ds}{dt}$$

The rate of change of the velocity...ie $\frac{\Delta v}{\Delta t}$

would represent??

$$acceleration = \frac{dv}{dt}$$

So it follows that the second derivative of displacement will give us acceleration:

$$a = \frac{d^2 s}{dt^2}$$ ←-------- Notice the notation

Example

If the displacement (in metres) at time t (in seconds) of an object is given by

$s = 4t^3 + 7t^2 - 2t,$

find the acceleration at time $t = 10$.

$S' = 12t^2 + 14t$ (velocity)

$S'' = 24t + 14$ (accel.)

$S''(10) = 24(10) + 14$

$= 254\,m/s^2$

Example:

- The position of a particle is given by the equation $s = f(t) = t^3 - 6t^2 + 9t$, where t is measured in seconds and s in meters.

  a) Find the velocity at time t.

  b) What is the velocity after 2 s? After 4 s?

  c) When is the particle at rest?

  d) When is the particle moving forward (that is, in the positive direction)?

  e) Draw a diagram to represent the motion of the particle.

  f) Find the total distance traveled by the particle during the first five seconds.

  g) Find the acceleration at time t and after 4 s.

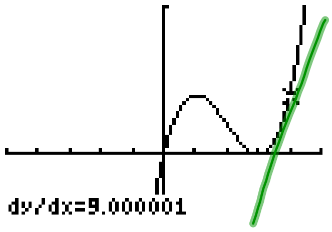
  h) Graph the position, velocity, and acceleration functions for $0 \leq t \leq 5$.

  i) When is the particle speeding up? When is it slowing down?

$s = f(t) = t^3 - 6t^2 + 9t$

a) Find the velocity at time t.

$$s' = 3t^2 - 12t + 9$$



b) What is the velocity after 2 s? After 4 s?

$$s'(2) = 3(2)^2 - 12(2) + 9$$
$$= -3 \text{ m/s}$$

$$s'(4) = 3(4)^2 - 12(4) + 9$$
$$= 9 \text{ m/s}$$

c) When is the particle at rest?

$s = f(t) = t^3 - 6t^2 + 9t$

$$\frac{3t^2}{3} - \frac{12t}{3} + \frac{9}{3} = \frac{0}{3}$$

$$t^2 - 4t + 3 = 0$$

$$(t-3)(t-1) = 0$$

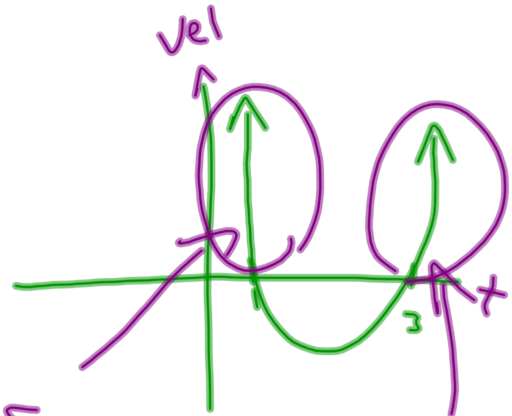
$t = 3$ sec OR $t = 1$ sec

velocity > 0

d) When is the particle moving forward (that is, in the positive direction)?

$$\frac{3t^2}{3} - \frac{12t}{3} + \frac{9}{3} > \frac{0}{3}$$

$$(t-3)(t-1) > 0$$



$0 \leq t < 1$

$t > 3$

e) Draw a diagram to represent the motion of the particle.

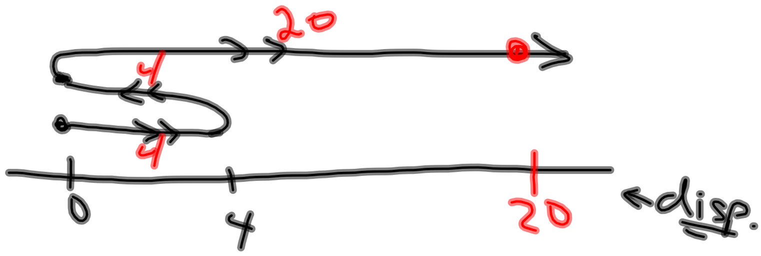
$s = f(t) = t^3 - 6t^2 + 9t$

start →

t	s
0	0
1	4
3	0
5	20

vel = 0 (t = 1, 3)

4, 4, 20



f) Find the total distance traveled by the particle during the first five seconds.

dist. = 4 + 4 + 20

= <u>28 m</u>

g) Find the acceleration at time t and after 4 s.

$s = f(t) = t^3 - 6t^2 + 9t$

$S'' = 6t - 12$

$S''(4) = 6(4) - 12$

$= \underline{12\,m/s^2}$

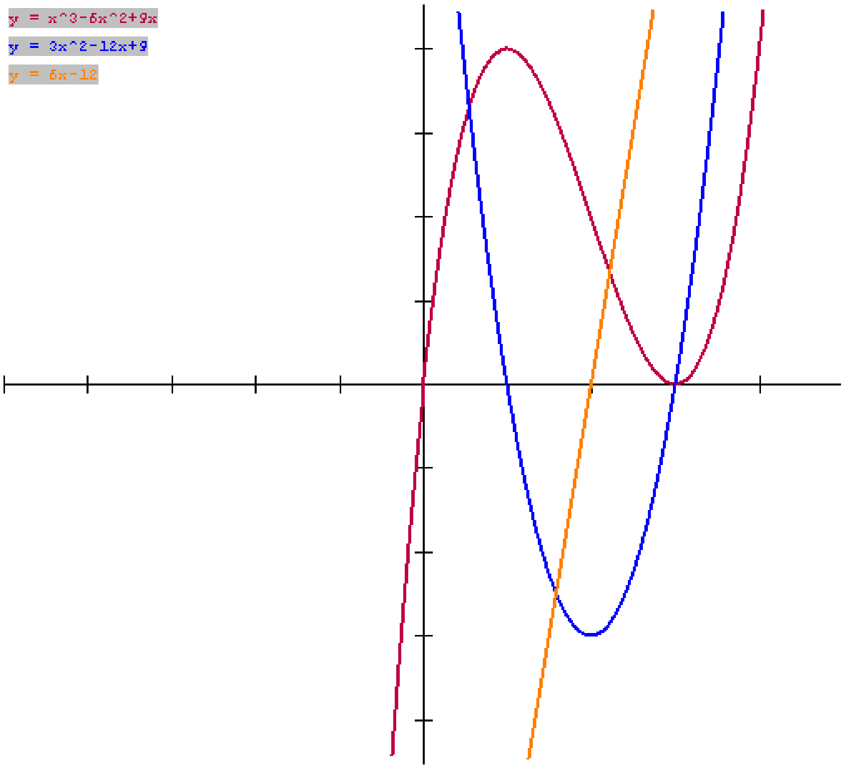
h) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.

i) When is the particle speeding up? When is it slowing down?

$s = f(t) = t^3 - 6t^2 + 9t$

$\boxed{\text{BONUS}}$

h) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.



i) When is the particle speeding up? When is it slowing down?