Warm Up

Evaluate each of the following limits. If they do not exist provide a reason.

$\sin^2 x + \cos^2 x = 1$

1. $\lim\limits_{x \to 0} \dfrac{1 - \cos^2 x}{5x^2}$

$\lim\limits_{x \to 0} \dfrac{\sin^2 x}{5x^2}$

$\lim\limits_{x \to 0} \left(\dfrac{\sin x}{x}\right)^2 \dfrac{1}{5}$

$= (1)^2 \dfrac{1}{5} = \dfrac{1}{5}$

2. $\lim\limits_{x \to 0} \dfrac{3x^3 - 6x^2}{\sin^2 4x}$

$\lim\limits_{x \to 0} \dfrac{3x^2(x-2)}{\sin^2 4x}$

$\lim\limits_{x \to 0} \left(\dfrac{4x}{\sin 4x}\right)^2 \dfrac{3(x-2)}{16}$

$= (1)^2 \dfrac{3(-2)}{16}$

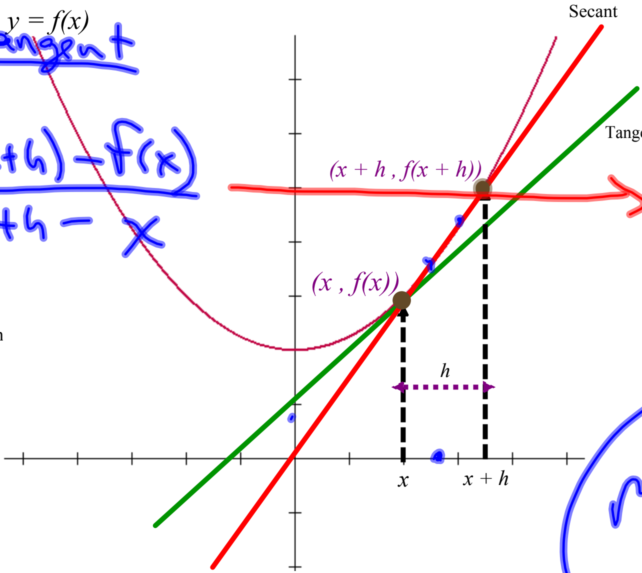
$= -\dfrac{6}{16} = \dfrac{-3}{8}$

Tangents, Velocities, and Rates of Change

Slope of a tangent to a curve:

Slope of tangent

$$m = \frac{f(x+h) - f(x)}{x+h - x}$$



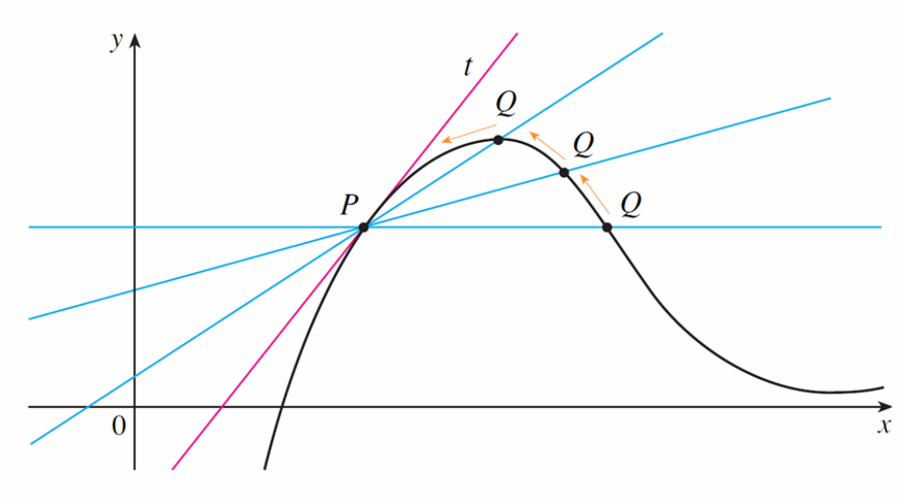
slope of tangent

$$\boxed{\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

How will we find the slope of a tangent drawn to a curve at a point *x*?

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

How will the slope of this secant become a better approximation for the slope of the tangent line?



Use your knowledge of limits to determine an expression for that would represent the slope of the tangent line drawn at the point *x*.

Example:

Determine the equation of the tangent line drawn to the curve $y = x^2 + 2$ at the point $x = -2$.

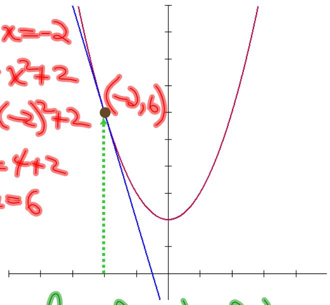
If $x=-2$

$y=x^2+2$

$y=(-2)^2+2$

$y=4+2$

$y=6$



Point-Slope Formula

$y-y_1=m(x-x_1)$

$y=mx+b$

$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ← what is the x?? → point of tangency

In general...

$f(x)=x^2+2$

$f(x+h)=(x+h)^2+2$

$=x^2+2xh+h^2+2$

$\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2\cancel{+2}-(\cancel{x^2}\cancel{+2})}{h}$

$\lim_{h\to 0}\frac{2xh+h^2}{h}$

$\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}$

Slope $=2x$

@ $x=-2$

$m=2(-2)$

$m=-4$

for specific point at $x=-2$

$f(-2)=(-2)^2+2$

$=6$

$f(-2+h)=(-2+h)^2+2$

$=4-4h+h^2+2$

$=6-4h+h^2$

$\lim_{h\to 0}\frac{(6-4h+h^2)-6}{h}$

$\lim_{h\to 0}\frac{-4h+h^2}{h}$

$\lim_{h\to 0}\frac{\cancel{h}(-4+h)}{\cancel{h}}$

$m=-4$

$\left.\begin{matrix} m=-4 \\ (-2,6)\end{matrix}\right\}$ $y-6=-4(x+2)$

$y-6=-4x-8$

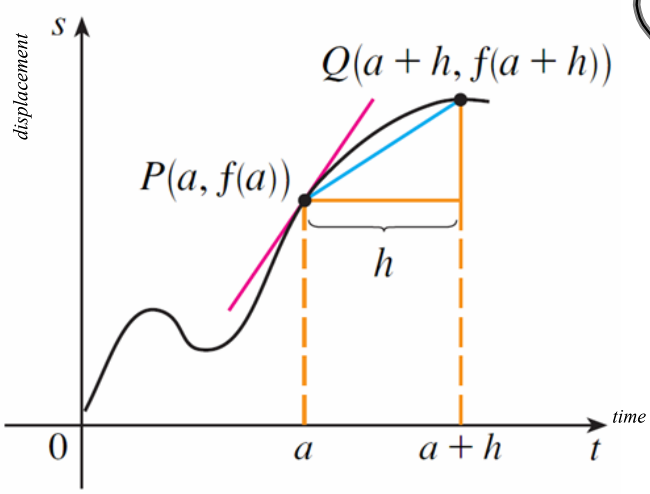
$4x+y+2=0$

or

$y=-4x-2$

What about other applications of limits?

Velocity...



$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{\Delta \text{disp}}{\Delta \text{time}}$$

velocity

$$m_{PQ} = \frac{f(a+h) - f(a)}{h}$$

$$= \text{average velocity}$$

FIGURE 6

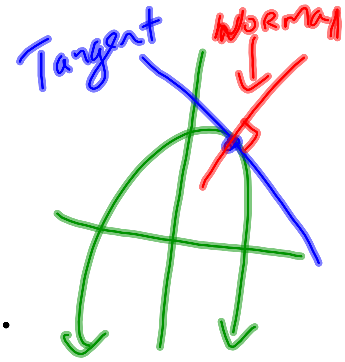
What if we were looking for instantaneous velocity?

?

- This means that the
  - velocity at time $t = a$ is equal to the
  - slope of the tangent line at P.

Ex.

Equation of a NORMAL to the Curve $y=\sqrt{3x+1}$ at $x=5$.



Point:

$y=\sqrt{3(5)+1}$

$y=4$

$(5,4)$

Slope

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$f(x)=\sqrt{3x+1}$

$f(x+h)=\sqrt{3(x+h)+1}$

$=\sqrt{3x+3h+1}$

$$\lim_{h\to 0}\frac{\sqrt{3x+3h+1}-\sqrt{3x+1}}{h}$$