Warm Up

Differentiate the following...

$$\left(x^3 + y^5\right)^6 + 3xy = 2x^4y^5$$

$$6(x^3+y^5)^5\left(3x^2+5y^4\frac{dy}{dx}\right)+3y+3x\frac{dy}{dx}=8x^3y^5+2x^4(5y^4)\frac{dy}{dx}$$

$$18x^2(x^3+y^5)^5+30y^4(x^3+y^5)^5\frac{dy}{dx}+3y+3x\frac{dy}{dx}=8x^3y^5+10x^4y^4\frac{dy}{dx}$$

$$\frac{\left(30y^4(x^3+y^5)^5+3x-10x^4y^4\right)\frac{dy}{dx}}{\left(30y^4(x^3+y^5)^5+3x-10x^4y^4\right)}=\boxed{\frac{8x^3y^5-18x^2(x^3+y^5)^5-3y}{30y^4(x^3+y^5)^5+3x-10x^4y^4}}$$

Higher Order Derivatives

We can continue to find the derivatives of a derivative. We find the

- second derivative by taking the derivative of the first,
- third derivative by taking the derivative of the second ... etc

Examples:

1. Determine the higher order derivatives for *f(x)*...

$$f(x) = x^4 - 2x^3 + 3x - 5$$

$$f'(x) = 4x^3 - 6x^2 + 3$$

$$f''(x) = 12x^2 - 12x$$

$$f'''(x) = 24x - 12$$

$$f''''(x) = 24$$

$$f'''''(x) = 0$$

2. Determine $f'''(x)$ given that $f(x) = \dfrac{5}{\sqrt{2-3x}}$

$$f(x) = 5(2-3x)^{-1/2}$$

$$f'(x) = -\frac{5}{2}(2-3x)^{-3/2}(-3)$$

$$f''(x) = \frac{15}{4}(2-3x)^{-5/2}(9) \rightarrow (-3)(-3)$$

$$f'''(x) = -\frac{75}{8}(2-3x)^{-7/2}(-27) \rightarrow (-3)(-3)(-3)$$

3. Find the second derivative of the implicit function $xy + y^2 = 4$.

$$y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-y}{x+2y} \qquad \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{dy}{dx}(x+2y) + y\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left(+\frac{y}{\cancel{x+2y}}\right)\cancel{(x+2y)} + y + 2y\left(\frac{-y}{x+2y}\right)}{(x+2y)^2}$$

$$= \frac{y + y - 2y\left(\frac{y}{x+2y}\right)}{(x+2y)^2}$$

$$= \frac{\left[2y - 2y\left(\frac{y}{\cancel{x+2y}}\right)\right]\cancel{x+2y}}{\left[(x+2y)^2\right](x+2y)}$$

$$= \frac{2y(x+2y) - 2y^2}{(x+2y)^3}$$

$$= \frac{2y\left[(x+2y) - y\right]}{(x+2y)^3}$$

$$= \frac{2y(x+y)}{(x+2y)^3}$$

4. *Determine the fourth derivative of* $y = \cos(5x)$

$y' = -\sin(5x)(5)$

$y'' = -\cos(5x)25$

$y''' = \sin(5x)125$

$y'''' = \cos(5x)(625)$

$y''''' = -\sin(5x)(3125)$

$\vdots$

$y^{(10)} = ?$

$y^{(10)} = -\cos(5x)(5)^{10}$

Practice…

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#2, 3, 4, 5, 7 (a)