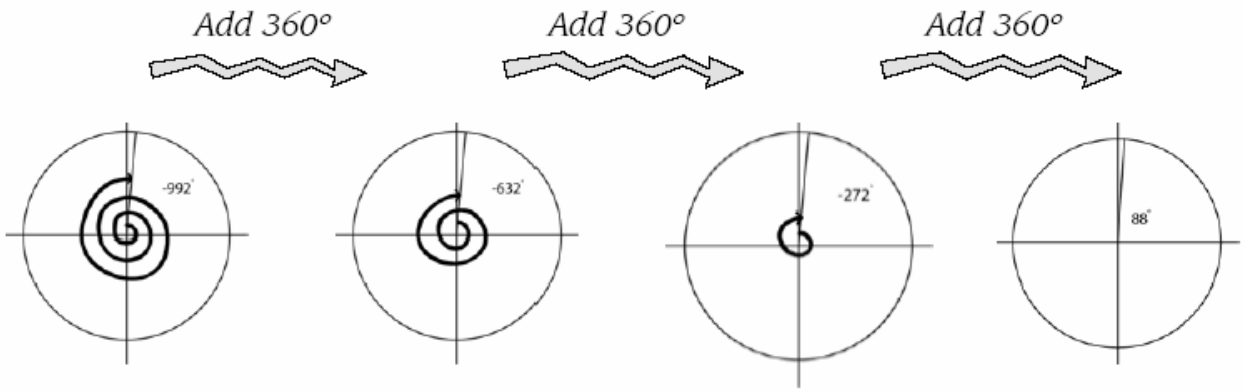
- Principal angle: is the **smallest positive** angle that describes the position of the terminal arm.

Boundary??? $\theta \leq$ principal angle $\leq 360^{o}$

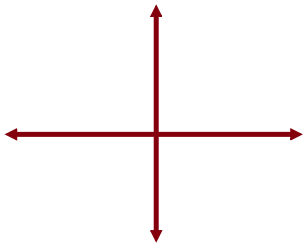
Example Given the co-terminal angle -992°, find the principal angle.

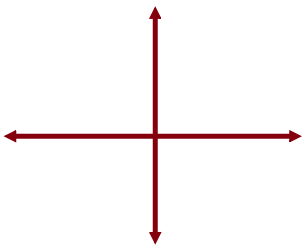
We need to "unwind" our way back to between 0° and 360° by making revolutions of 360°.



The principal angle is 88°

Examples... a) -260^{o} b) 680^{o}





What about a strategy for much larger angles??

Find the principal angle for the following angles:

(1) 134 723° = 83°

```
134723/360
      374.2305556
Ans-374
      .2305555556
Ans*360
                  83
```

(2) -34 211°

```
      -95.03055556
Ans+95
      -.0305555556
Ans*360
                -11
Ans+360
                349
```

Must be positive

co-terminal

(3) 5 345 781°

```
5345781/360
      14849.39167
Ans-14849
      .391666667
Ans*360
      141.0000001
```

-141°

(4) -278 153°

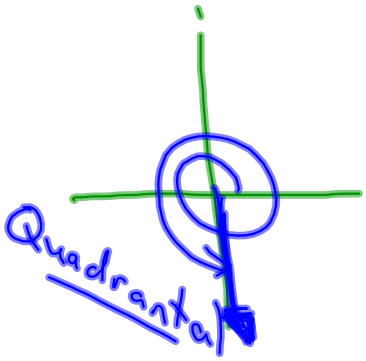
```
      -772.6472222
Ans+772
      -.6472222222
Ans*360
                -233
Ans+360
                 127
```

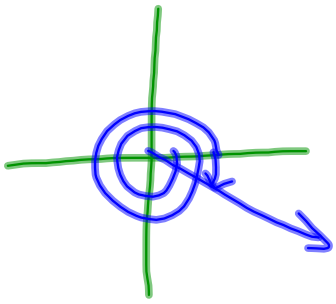
127°

Warm Up

1. Sketch each of the following:

(a) 630^o (b) -740^o





2. Determine the principal angle for each of the following:
(a) $13\ 679^o$ (b) $-376\ 895^o$

PA = 359°

PA = 25°

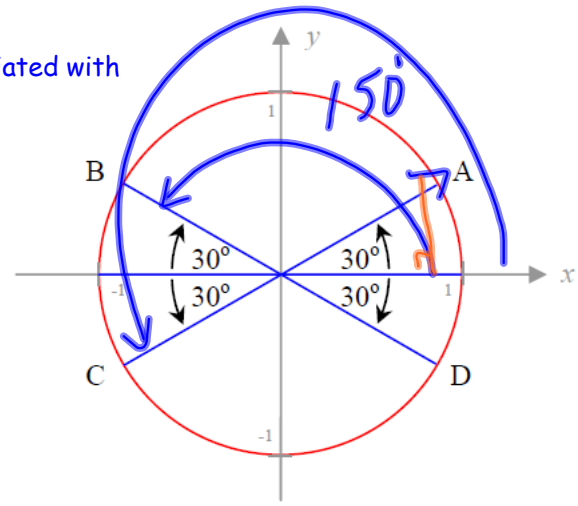
Reference Angles

Definition of Reference Angle

When an angle is drawn in standard position, its reference angle is the positive acute angle measured from x-axis to the angle's terminal side.

What rotation angles would be associated with

a 30° reference angle?

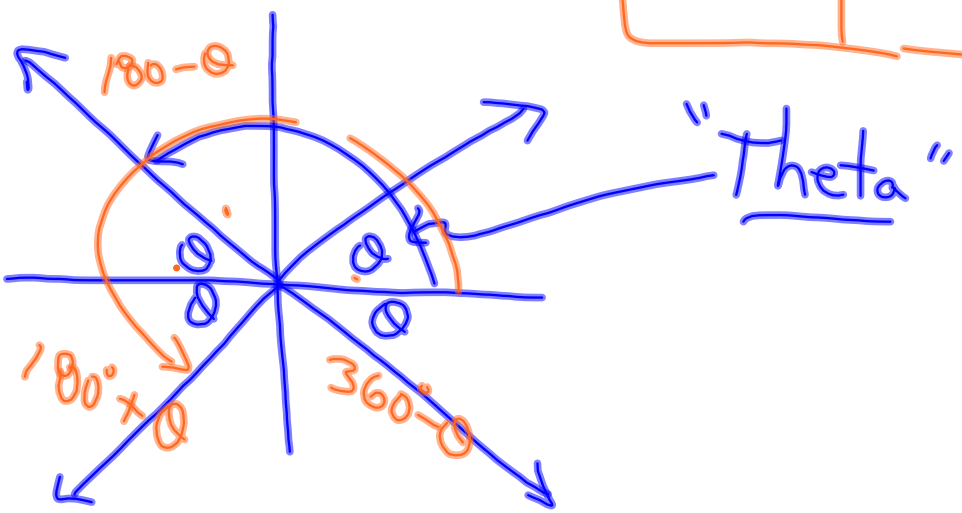


Determining rotation angle from a known reference angle ...

Let's develop a rule for each of the quadrants...

$180°-\theta$	θ
$180°+\theta$	$360°-\theta$

Reference Angle = θ



Complete the chart shown below...

Reference Angle	Quadrant	Rotation Angle
24°	3	204°
48°	2	132°
75°	4	285°
80°	1	80°

Determine the reference angle associated with each of the following rotation angles...

Rotation Angle	Reference Angle
325°	35°
-174°	6°
1240°	20°

RA = 160°

Practice problems...

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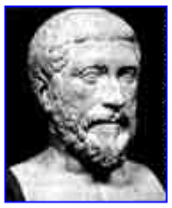
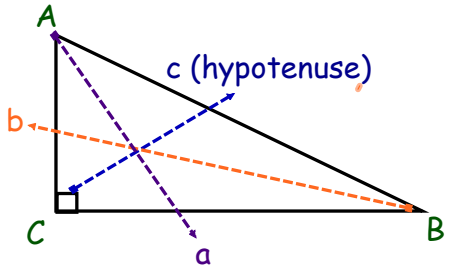
#2, 3, 4, 5, 6, 7



Pythagorean Theorem

- is a fundamental relationship amongst the sides on a **RIGHT triangle.**



FORMULA???

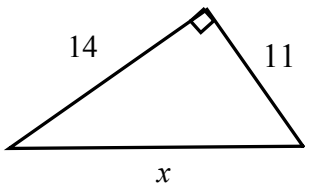
$$c^2 = a^2 + b^2$$

OPTIONS...

#1. Finding the unknown hypotenuse:

$$c^2 = a^2 + b^2$$

ex:



$x^2 = 14^2 + 11^2$

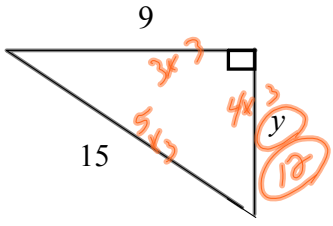
$\sqrt{x^2} = \sqrt{317}$

$x = \sqrt{317}$

#2. Finding an unknown side

$$a^2 = c^2 - b^2$$

ex:



$y^2 + 9^2 = 15^2$

$y^2 = 15^2 - 9^2$

$\sqrt{y^2} = \sqrt{144}$

$y = 18$

Pythagorean Triples...

3 - 4 - 5

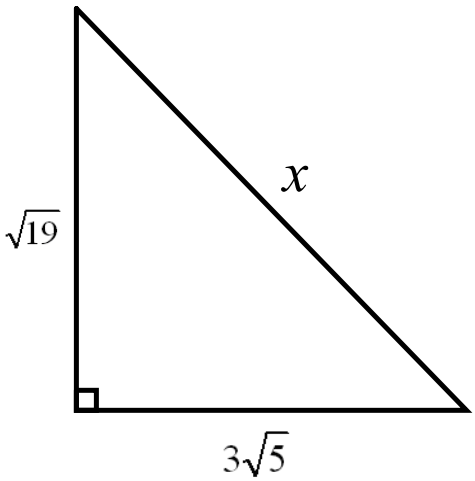
5 - 12 - 13

#1

#2

Check Up...

Determine the measure of the variable in each of the following diagrams:

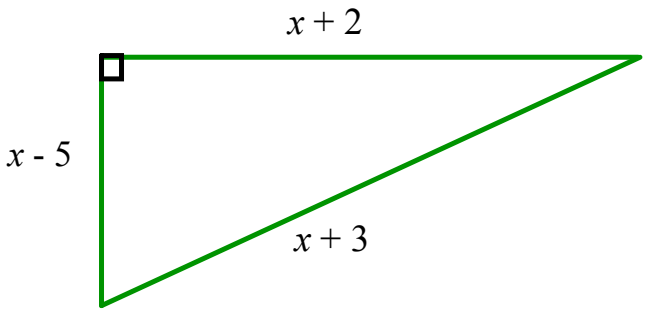


$x^2 = (\sqrt{19})^2 + (3\sqrt{5})^2$

$x^2 = 19 + 45$

$\sqrt{x^2} = \sqrt{64}$

$x = 8$



$(x+2)^2 + (x-5)^2 = (x+3)^2$

$x^2 + 4x + 4 + x^2 - 10x + 25 = x^2 + 6x + 9$

$x^2 - 12x + 20 = 0$

$(x-10)(x-2) = 0$

$x = 10, 2$ — Inadmiss. b/c $(x-5)$