Part II : Open Response [40 MARKS]

Show all work for each of the following in the space provided.

1. Expand and simplify the following radical expression: [8]

$$\left(2\sqrt{54}-\sqrt{8}\right)^2-\left(5\sqrt{18}-3\sqrt{24}\right)\left(4\sqrt{50}-\sqrt{27}\right)$$

$$\left(6\sqrt{6}-2\sqrt{2}\right)^2 \cdots -$$

$$36(6)-24\sqrt{12}+8$$

$$\boxed{\text{Ans: } 45\sqrt{6}+192\sqrt{3}-54\sqrt{2}-376}$$

2. (a) Solve: $\sqrt{5x-1}-\sqrt{x+2}=1$ [6]

$$\left(1+\sqrt{x+2}\right)^2$$

$$\Downarrow$$

$$4x^2-9x+2=0$$

$$\Downarrow$$

$$(4x-1)(x-2)=0$$

Verify:

$x=\frac{1}{4}$ extraneous $\quad \boxed{x=2}$ ✓

(b) Solve the equation $\frac{4m-1}{m+2}-\frac{m+1}{m-2}=\frac{m^2-4m+24}{m^2-4}$. State all non-permissible values of m. [6]

$$\cancel{(m+2)}(m-2)\frac{4m-1}{\cancel{m+2}}-(m+2)\cancel{(m-2)}\frac{m+1}{\cancel{m-2}}=\frac{m^2-4m+24}{(m-2)\cancel{(m+2)}}\cancel{(m+2)}\cancel{(m-2)}$$

$$(m-2)(4m-1)-(m+1)(m+2)=m^2-4m+24$$

$$\Downarrow$$

$$m^2-4m-12=0$$

$$(m-6)(m+2)=0$$

$$m=6,\cancel{-2}$$

$$m\neq\pm2$$

3. Rationalize the denominator and simplify each of the following radical expressions: [6]

(a) $\dfrac{3w^5}{x^3}\sqrt[3]{\dfrac{4x^8}{10w^6}}$

$\dfrac{3w^5}{x^3}\left(\dfrac{\sqrt[3]{4x^8}}{\sqrt[3]{10w^6}}\right)$

$\left(\dfrac{3w^5}{x^3}\right)\dfrac{x^2\sqrt[3]{4x^2}}{w^2\sqrt[3]{10}}$

$\dfrac{3x^2w^5}{x^3w^2}\left(\dfrac{\sqrt[3]{4x^2}}{\sqrt[3]{10}}\right)\left(\dfrac{\sqrt[3]{10^2}}{\sqrt[3]{10^2}}\right) =$

$\dfrac{3x^2w^5\sqrt[3]{400x^2}}{10x^3w^2}$

$= \dfrac{6x^2w^5}{10x^3w^2}\sqrt[3]{50x^2} = \boxed{\dfrac{3w^3}{5x}\sqrt[3]{50x^2}}$

(b) $\dfrac{5-3\sqrt{6}}{3\sqrt{8}+1}\left(\dfrac{3\sqrt{8}-1}{3\sqrt{8}-1}\right)$

$\boxed{= \dfrac{30\sqrt{2}-5-36\sqrt{3}+3\sqrt{6}}{71}}$

4. Simplify the following rational expression. State all non-permissible values of the variable. [6]

$$\left(\frac{x+5}{x^2-6x-27}-\frac{x-5}{x^2-11x+18}\right)\times\frac{4-x^2}{10x+10}\div\frac{2x+4}{x^2-9}$$

$x \neq 9, -3, 2, -1, -2, 3$

$\left[\dfrac{x+5}{(x-9)(x+3)}-\dfrac{x-5}{(x-9)(x-2)}\right] \times \dfrac{-1(\cancel{2-x})(\cancel{2+x})}{\cancel{10}_{2}(\cancel{x+1})} \cdot \dfrac{(x-3)(\cancel{x+3})}{2(\cancel{x+2})}$

$\dfrac{(x+5)(x-2)-(x-5)(x+3)}{(x-9)(x+3)(x-2)}$

$5x+5$

$\dfrac{\cancel{5}(\cancel{x+1})}{(x-9)(\cancel{x+3})(\cancel{x-2})} = \dfrac{-1(x-3)}{4(x-9)}$

5. A speedboat can travel 32 miles per hour in still water. It travels 150 miles upstream against the current then returns to the starting location. The total time of the trip is 10 hours. What is the speed of the current? [6]

x → current	d (mi)	s (mph)	t (h)
with	150	x+32	$\frac{150}{x+32}$
against	150	32−x	$\frac{150}{32-x}$

$t = \dfrac{d}{s}$

$\dfrac{150}{x+32}+\dfrac{150}{32-x}=10$

$x^2=64$

$x = 8$

M/choice

#4) $\dfrac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)}{\left(\frac{1}{a}+\frac{1}{b}\right)}$ $\dfrac{a^2b^2}{a^2b^2}$

$\dfrac{b^2-a^2}{ab^2+a^2b}$

$\dfrac{(b-a)\cancel{(b+a)}}{ab\cancel{(b+a)}}$

#9) $\dfrac{-1}{w+1}-\dfrac{w+1}{w-1}$

$\dfrac{-1(w-1)-(w+1)(w+1)}{(w+1)(w-1)}$

$\dfrac{-w+1-(w^2+2w+1)}{(w+1)(w-1)}$

$=\dfrac{-w^2-3w}{w^2-1}$

$=\dfrac{-w(w+3)}{w^2-1}$

12) $\dfrac{x+2}{\sqrt[4]{6-2x}}$

$6-2x>0$

$-2x>-6$

$x<3$

14) $\dfrac{u}{3v}+\dfrac{u}{4v}$

$\dfrac{4uv+3uv}{12v^2}$

$\dfrac{\cancel{v}(4u+3u)}{12v^{\cancel{2}}}$

$=\dfrac{7u}{12v}$

$\dfrac{4u+3u}{12v}$

$\dfrac{7u}{12v}$

15) $\sqrt{2x+15} - x = 6$

$(\sqrt{2x+15})^2 = (6+x)^2$

$2x+15 = 36 + 12x + x^2$

$0 = x^2 + 10x + 21$

$0 = (x+7)(x+3)$

$x = -7, -3 \rightarrow \sqrt{-14+15} - (-7) \mid 6$

extraneous

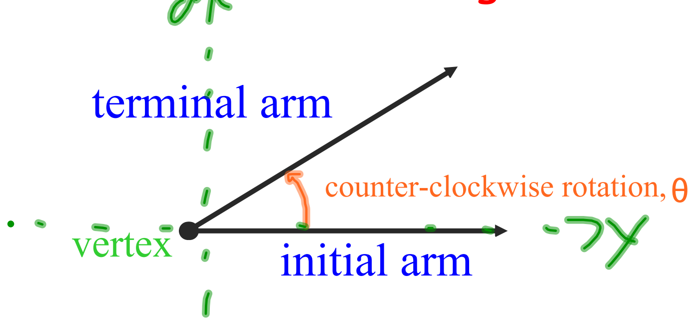
$1 + 7 \neq 6$

McGraw-Hill Ryerson

Pre-Calculus 11

Chapter 2
Trigonometry

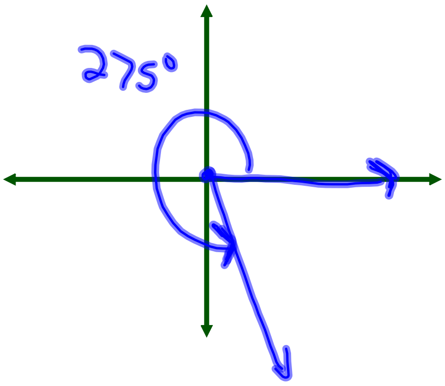
Rotation Angles



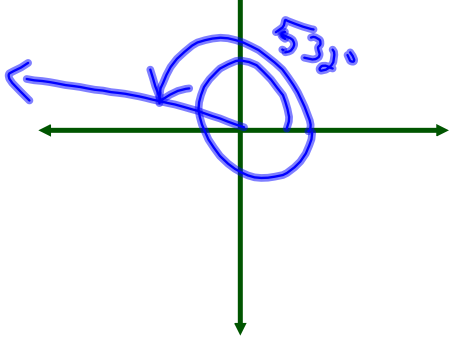
- **standard position** - when the initial arm is on the **positive** *x*-axis and the vertex is at the origin.

ex: positive rotation - counter clockwise (ccw)

#1: 275°

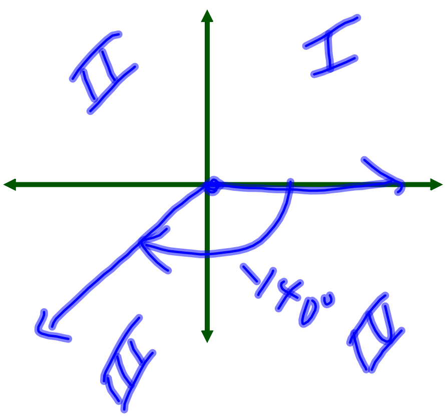


#2: 532° − 360° = 172°

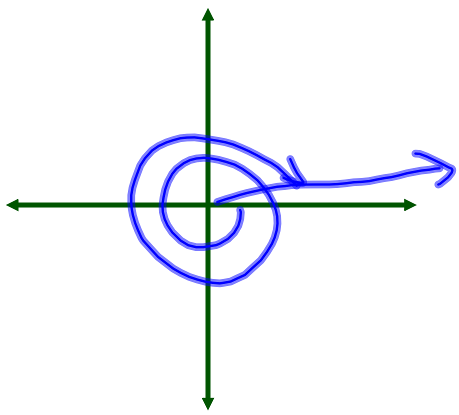


ex: negative rotation - clockwise (cw)

#3: -140°



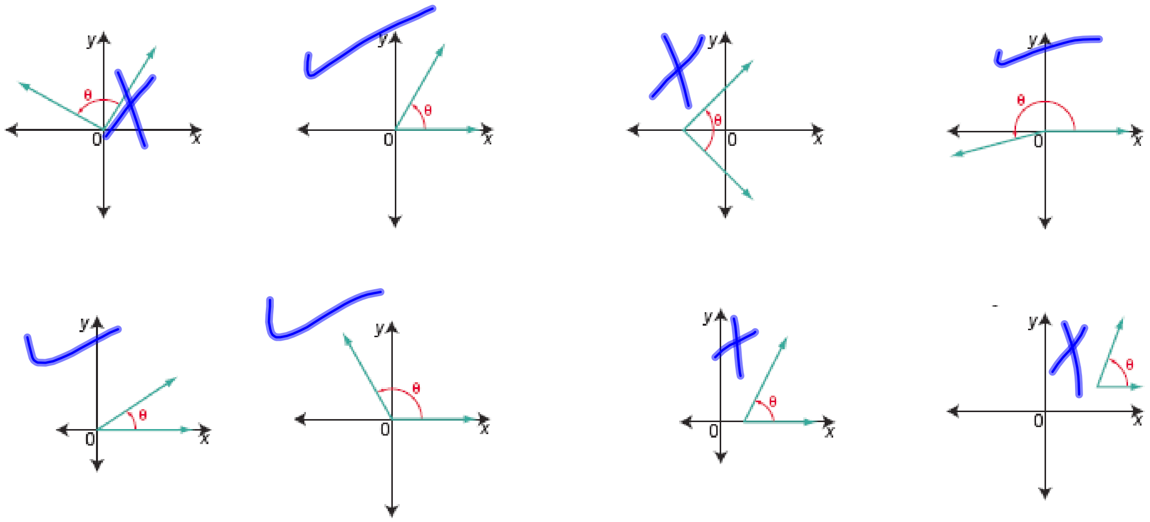
#4: -714°



Chapter
2

Angles in Standard Position

Circle the angles that are in standard position.

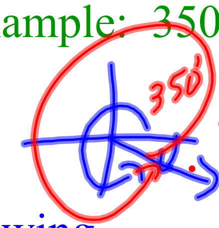


Check answer

- **co-terminal angles** - angles with the same terminal arm

$\theta + 360^{o}k$, where k is an integer

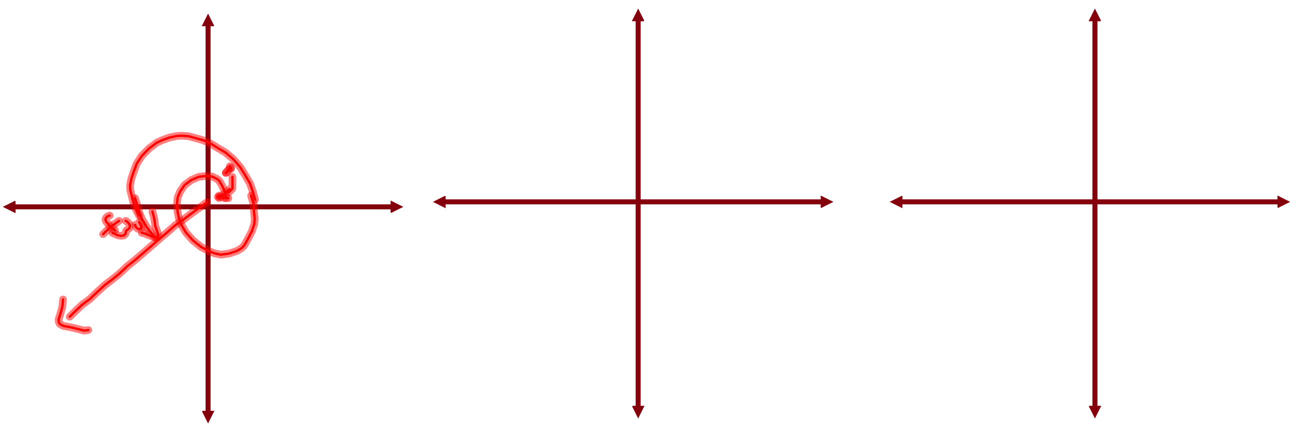
example: 350^{o} -360° -10° $\times 360$ 710° 1070°



HOW?

EXERCISE: Sketch each of the following...

a) 583^{o} $\rightarrow 223^{\circ}$ b) -235^{o} $\Rightarrow 125$ c) -810^{o}



Identify 3 angles co-terminal with each of the following angles:

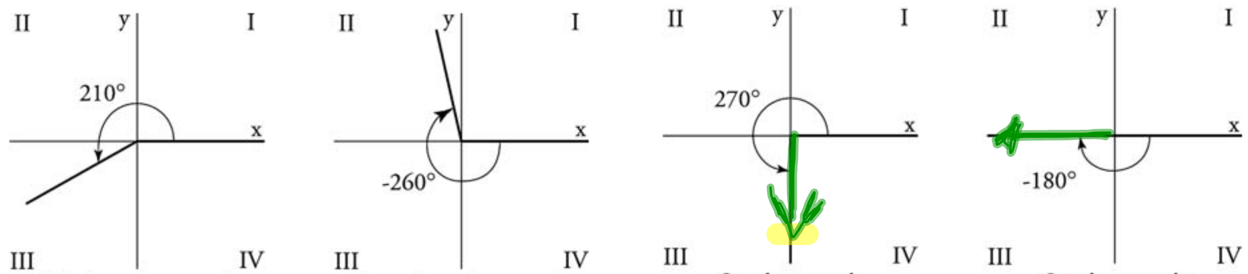
(1) 47°

(2) -453°

How many co-terminal angles exist for any rotation angle?

Infinite (∞)

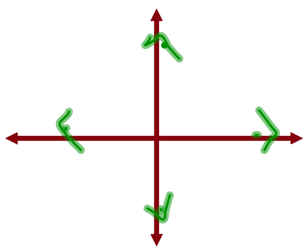
Angles that are in standard position are said to be **quadrantal** if their terminal side coincides with a coordinate axis. Angles in standard position that are not quadranta fall in one of the four quadrants, as shown below...



Third quadrant angle (a) | Second quadrant angle (b) | Quadrant angle (e) | Quadrant angle (f)

- <u>Quadrantal angle</u>: terminal arm lies on a quadrant boundary (axis)

examples...



Within which quadrant would the terminal arm for each of the following rotation angles be found?

$94°$ II

$500°$ III

$-100°$

$180°$

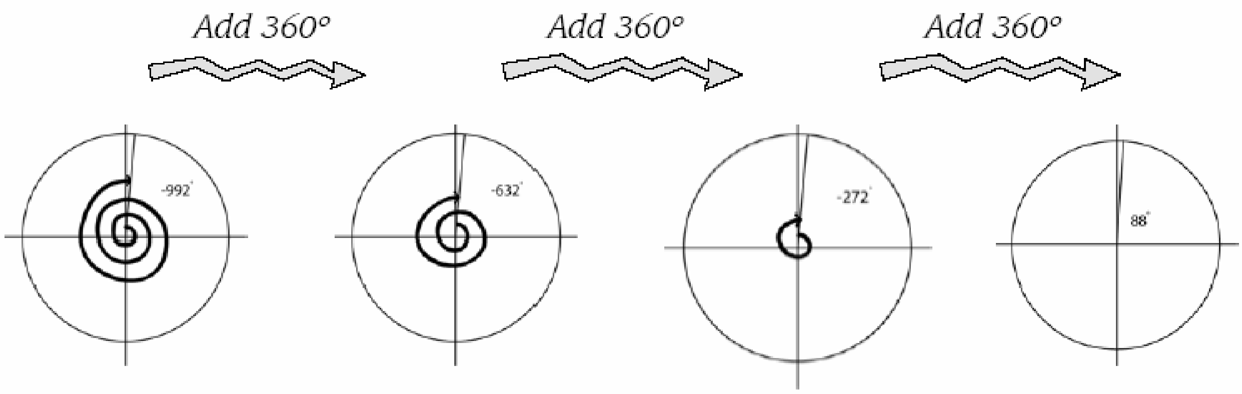
$-300°$

- Principal angle: is the **smallest positive** angle that describes the position of the terminal arm.

Boundary??? $\theta \leq$ principal angle $\leq 360^{\circ}$

Example Given the co-terminal angle -992°, find the principal angle.

We need to "unwind" our way back to between 0° and 360° by making revolutions of 360°.



The principal angle is 88°

Examples... a) -260° b) 680°