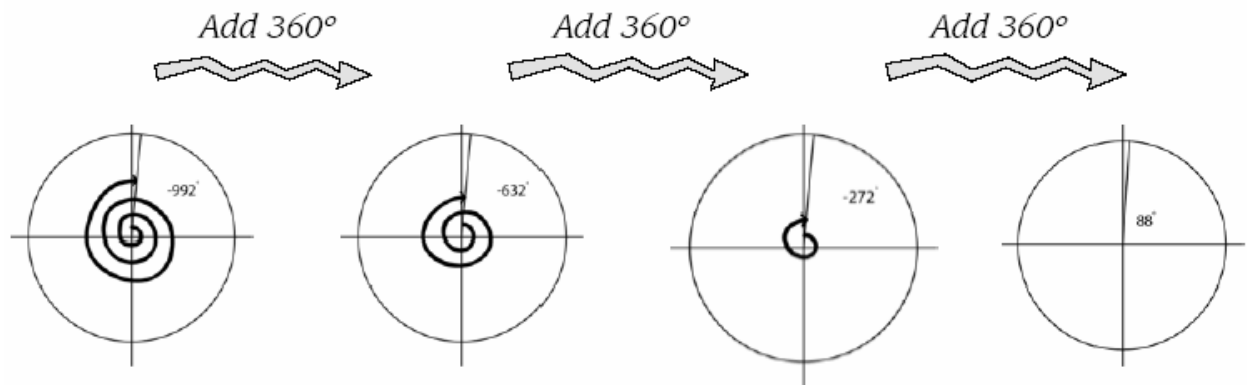


- Principal angle: is the **smallest positive** angle that describes the position of the terminal arm.

Boundary??? $\theta \leq \text{principal angle} \leq 360^\circ$

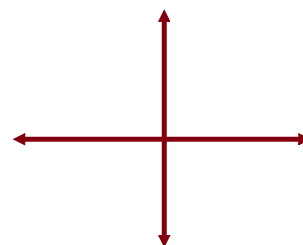
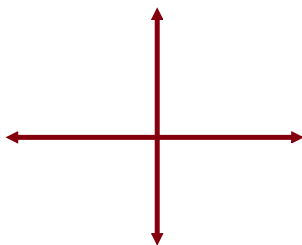
Example ... Given the co-terminal angle -992° , find the principal angle.

We need to "unwind" our way back to between 0° and 360° by making revolutions of 360° .



The principal angle is 88°

Examples... a) -260° b) 680°



What about a strategy for much larger angles??

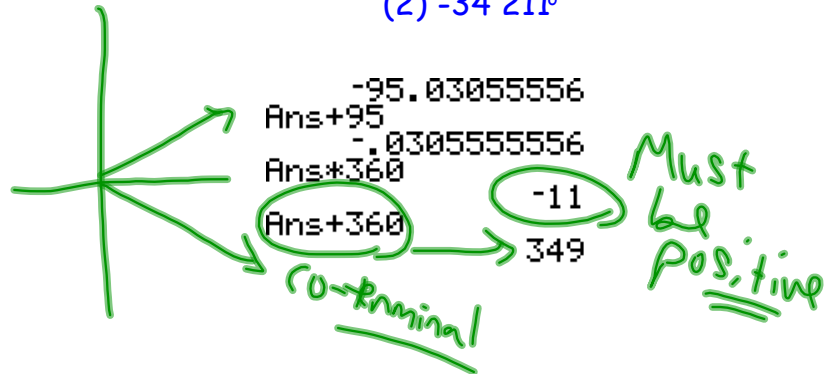
Find the principal angle for the following angles:

(1) $134\ 723^\circ = 83^\circ$

```

134723/360
374.2305556
Ans-374
.230555556
Ans*360
83
    
```

(2) $-34\ 211^\circ$



(3) $5\ 345\ 781^\circ$

```

5345781/360
14849.39167
Ans-14849
.391666667
Ans*360
141.0000001
    
```

141°

(4) $-278\ 153^\circ$

```

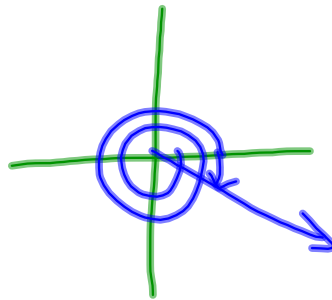
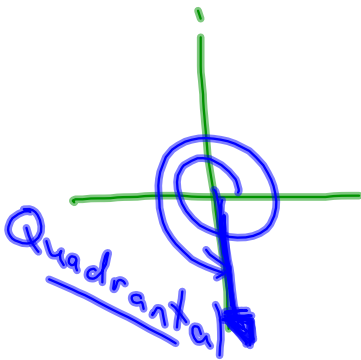
-772.6472222
Ans+772
-.647222222
Ans*360
-233
Ans+360
127
    
```

127°

Warm Up

1. Sketch each of the following:

(a) 630° (b) -740°



2. Determine the principal angle for each of the following:

(a) $13\,679^\circ$ (b) $-376\,895^\circ$

$$PA = \underline{359^\circ}$$

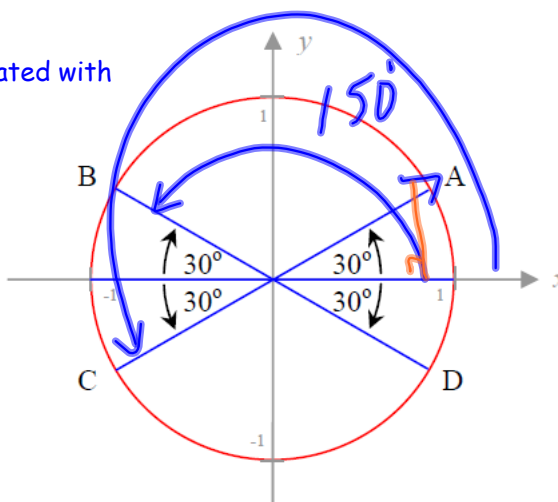
$$PA = \underline{25^\circ}$$

Reference Angles

Definition of Reference Angle

When an angle is drawn in standard position, its reference angle is the positive acute angle measured from x-axis to the angle's terminal side.

What rotation angles would be associated with a 30° reference angle?

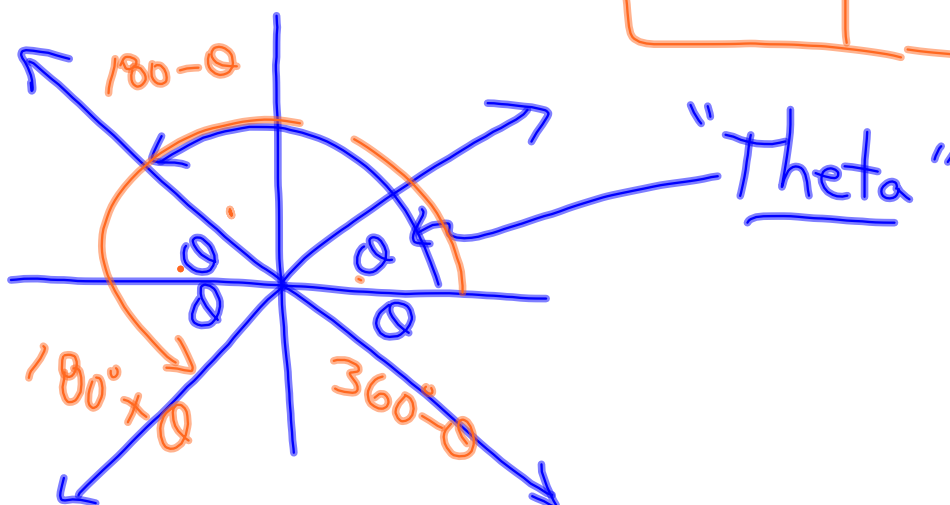


Determining rotation angle from a known reference angle ...


Let's develop a rule for each of the quadrants...

Reference Angle = θ

$180^\circ - \theta$	θ
$180^\circ + \theta$	$360^\circ - \theta$




Complete the chart shown below...



Reference Angle	Quadrant	Rotation Angle
24°	3	204°
48°	2	132°
75°	4	285°
80°	1	80°

Determine the reference angle associated with each of the following rotation angles...



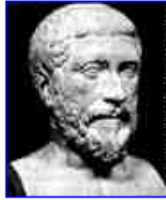
Rotation Angle	Reference Angle
325°	35°
-174°	6°
<u>1240°</u>	20°

PA = 160°

Practice problems...

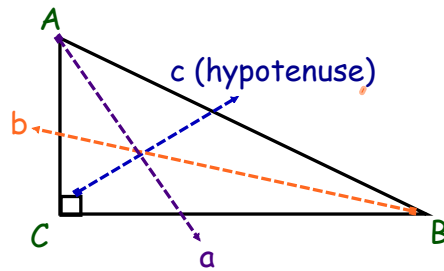
Pg. 83

#2, 3, 4, 5, 6, 7



Pythagorean Theorem

- is a fundamental relationship amongst the sides on a **RIGHT triangle**.



FORMULA???

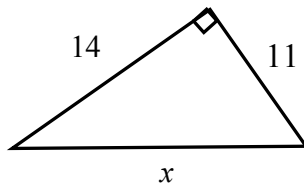
$$c^2 = a^2 + b^2$$

OPTIONS...

#1. Finding the unknown hypotenuse:

$$c^2 = a^2 + b^2$$

ex:

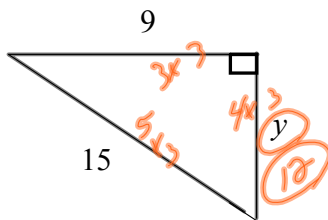


$$\begin{aligned} x^2 &= 14^2 + 11^2 \\ \sqrt{x^2} &= \sqrt{317} \\ x &= \sqrt{317} \end{aligned}$$

#2. Finding an unknown side

$$a^2 = c^2 - b^2$$

ex:



$$\begin{aligned} y^2 + 9^2 &= 15^2 \\ y^2 &= 15^2 - 9^2 \\ \sqrt{y^2} &= \sqrt{144} \\ y &= 12 \end{aligned}$$

Pythagorean Triples...

3 - 4 - 5

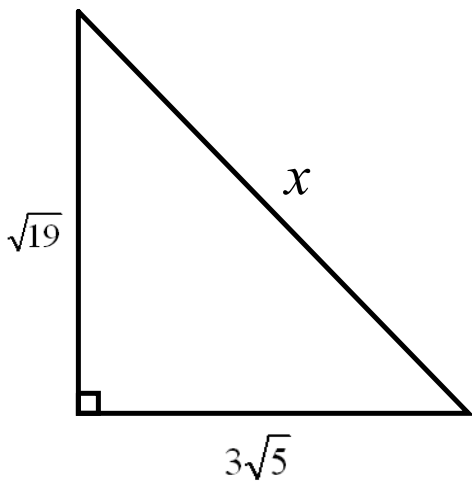
5 - 12 - 13

#1

#2

Check Up...

Determine the measure of the variable in each of the following diagrams:

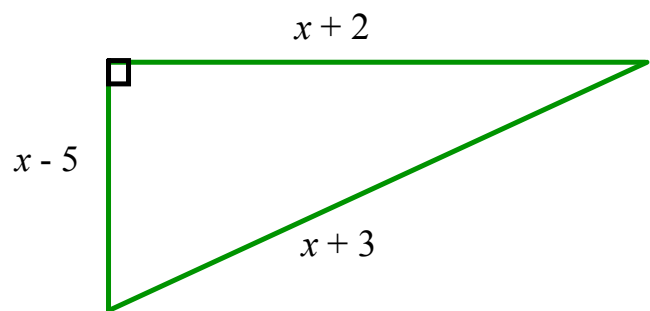


$$x^2 = (\sqrt{19})^2 + (3\sqrt{5})^2$$

$$x^2 = 19 + 45$$

$$\sqrt{x^2} = \sqrt{64}$$

$$x = 8$$



$$(x+2)^2 + (x-5)^2 = (x+3)^2$$

$$x^2 + 4x + 4 + x^2 - 10x + 25 = x^2 + 6x + 9$$

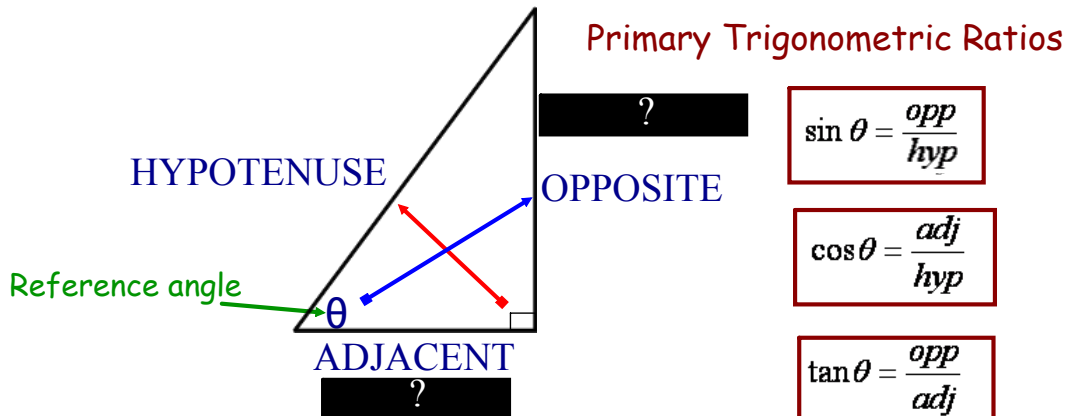
$$x^2 - 12x + 20 = 0$$

$$(x-10)(x-2) = 0$$

$$x = 10, 2$$

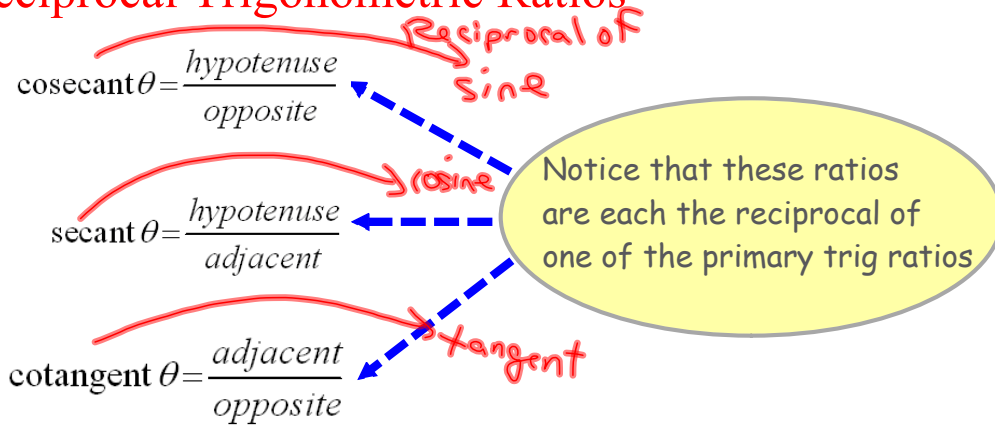
$x = 2$ is inadmissible because it would make the vertical leg $x-5$ negative.

Trigonometric Ratios



Memory Aid: "SOH CAH TOA"

Reciprocal Trigonometric Ratios



Summary	
Primary Ratios	Reciprocal Ratios
$\sin \theta = \frac{\text{opp}}{\text{hyp}}$	$\csc \theta = \frac{\text{hyp}}{\text{opp}}$
$\cos \theta = \frac{\text{adj}}{\text{hyp}}$	$\sec \theta = \frac{\text{hyp}}{\text{adj}}$
$\tan \theta = \frac{\text{opp}}{\text{adj}}$	$\cot \theta = \frac{\text{adj}}{\text{opp}}$