

Rational Expression Exam

1. Simplify the following

$$\text{a) } \frac{27x^4y^2}{-81x^2y^7} = \frac{3^3x^4y^2}{-3^4x^2y^7} = -\frac{x^2}{3y^5} \qquad \text{b) } \frac{15(x-2)(x+3)^2}{6(x+3)(x+2)} = \frac{5(x-2)(x+3)}{2(x+2)}$$

$$\text{c) } \frac{9x^2 + 6xy - 3y^2}{12x^2 - 12y^2} = \frac{3(3x^2 + 2xy - y^2)}{12(x^2 - y^2)} = \frac{3(3x - y)(x + y)}{3 \cdot 2^2(x + y)(x - y)} = \frac{(3x - y)}{2^2(x - y)}$$

$$\text{d) } \frac{x^3 + 5x^2 + 6x}{x^2 - 4} = \frac{x(x^2 + 5x + 6)}{x^2 - 4} = \frac{x(x+3)(x+2)}{(x-2)(x+2)} = \frac{x(x+3)}{(x-2)}$$

2. Multiplication and Division

$$\text{a) } \frac{4x^2z}{15y^4} \cdot \frac{25y^6}{16x^7} = \frac{2^2 \cdot 5^2 x^2 y^6 z}{2^4 \cdot 3 \cdot 5x^7 y^4} = \frac{5y^2z}{2^2 \cdot 3x^5}$$

$$\text{b) } \frac{x^2 - 9}{x^2 - 6x + 9} \cdot \frac{x^2 - 2x - 3}{(x+3)} = \frac{(x+3)(x-3)(x-3)(x+1)}{(x-3)(x-3)(x+3)} = (x+1)$$

$$\text{c) } \frac{(x+2)^2}{2x} \cdot \frac{x^2 - 3x}{x^2 - 5x + 6} = \frac{(x+2)^2 \cdot x \cdot (x-3)}{2x \cdot (x-3)(x-2)} = \frac{(x+2)^2}{2(x-2)}$$

$$\text{d) } \frac{x+2}{x-2} \cdot \frac{x^2 - 4}{x^2 + x - 2} = \frac{(x+2)(x+2)(x-2)}{(x-2)(x+2)(x-1)} = \frac{(x+2)}{(x-1)}$$

$$\text{e) } \frac{10m^2n}{6m-9} \div \frac{25mn^2}{2m-3} = \frac{2 \cdot 5m^2n \cdot (2m-3)}{2(2m-3) \cdot 5^2mn^2} = \frac{m}{5n}$$

$$\text{f) } \frac{x^2 + 5x + 6}{x^2 - 5x + 6} \div \frac{x^2 - x - 6}{x^2 + x - 6} = \frac{(x+3)(x+2)(x+3)(x-2)}{(x-3)(x-2)(x-3)(x+2)} = \frac{(x+3)^2}{(x-3)^2}$$

$$\text{g) } \frac{3x^2 + 8x + 4}{9x^2 - 4} \div \frac{2x^2 + 5x + 2}{3x^2 - 5x + 2} = \frac{(3x+2)(x+2)(3x-2)(x-1)}{(3x+2)(3x-2)(2x+1)(x+2)} = \frac{(x-1)}{(2x+1)}$$

$$\text{h) } \frac{10a + 8 - 3a^2}{a^2 - a - 12} \cdot \frac{9a^3 - 81a}{3a^2 - 7a - 6} = \frac{-1(3a+2)(a-4)9a(a+3)(a-3)}{(a-4)(a+3)(3a+2)(a-3)} = -9a$$

3. Addition and Subtraction

$$a) \frac{2x^2 + 3}{x - 3} + \frac{x^2 - 2}{x - 3} = \frac{3x^2 + 1}{(x - 3)} = \frac{(3x^2 + 1)}{(x - 3)}$$

$$b) \frac{1}{a - x} - \frac{3x}{a^2 - x^2} - \frac{a}{ax + x^2} = \frac{1}{(a - x)} - \frac{3x}{(a + x)(a - x)} - \frac{a}{x(a + x)} \Rightarrow LCD = x(a + x)(a - x)$$

$$\frac{1 \cdot x \cdot (a + x) - 3x \cdot x - a \cdot (a - x)}{x(a + x)(a - x)} = \frac{ax + x^2 - 3x^2 - a^2 + ax}{x(a + x)(a - x)} = \frac{-2x^2 + 2ax - a^2}{x(a + x)(a - x)}$$

$$c) \frac{3x^2 + 4}{x - 5} + \frac{x^2 - 7}{5 - x} = \frac{3x^2 + 4}{x - 5} - \frac{x^2 - 7}{x - 5} = \frac{3x^2 + 4 - x^2 + 7}{x - 5} = \frac{2x^2 + 11}{x - 5}$$

$$\frac{1}{2x} + \frac{5x}{x^2 - 1} + \frac{3}{x + 1} = \frac{1}{2 \cdot x} + \frac{5x}{(x + 1)(x - 1)} + \frac{3}{(x + 1)} \Rightarrow LCD = 2x(x + 1)(x - 1)$$

$$d) \frac{1 \cdot (x + 1)(x - 1) + 5x \cdot 2x + 3 \cdot 2x(x - 1)}{2x(x + 1)(x - 1)} = \frac{x^2 - 1 + 10x^2 + 6x^2 - 6x}{2x(x + 1)(x - 1)} =$$

$$\frac{17x^2 - 6x - 1}{2x(x + 1)(x - 1)}$$

$$\frac{2}{x - 4} + \frac{2x + 3}{x^2 - 5x + 4} = \frac{2}{x - 4} + \frac{2x + 3}{(x - 4)(x - 1)} \Rightarrow LCD = (x - 4)(x - 1)$$

$$e) \frac{2(x - 1) + (2x + 3)}{(x - 4)(x - 1)} = \frac{2x - 2 + 2x + 3}{(x - 4)(x - 1)} = \frac{4x + 1}{(x - 4)(x - 1)}$$

$$\frac{x + 5}{x^2 - 5x - 36} + \frac{x - 6}{x^2 - 11x + 18} = \frac{x + 5}{(x - 9)(x + 4)} + \frac{x - 6}{(x - 9)(x - 2)} \Rightarrow LCD = (x - 9)(x + 4)(x - 2)$$

$$f) \frac{(x + 5)(x - 2) + (x - 6)(x + 4)}{(x - 9)(x + 4)(x - 2)} = \frac{x^2 + 3x - 10 + x^2 - 2x - 24}{(x - 9)(x + 4)(x - 2)} = \frac{2x^2 + x - 34}{(x - 9)(x + 4)(x - 2)}$$

4. For each of the given expressions determine:

- the values of "x" which will make the expression equal to zero (this occurs when there is a value of "x" that makes the numerator equal to zero)
- the values of "x" for which the function would be undefined (occurs when there is a value of "x" that makes the denominator equal to zero)

$$\frac{8x+12}{4x-20}$$

$$a) a) 8x+12=0 \Rightarrow 8x=-12 \Rightarrow x=\frac{-12}{8}=\frac{-3}{2}$$

$$b) 4x-20=0 \Rightarrow 4x=20 \Rightarrow x=5$$

$$\frac{2d^3+4d^2-16d}{d^2+d-12}$$

$$a) 2d(d+4)(d-2)=0$$

$$\Rightarrow 2d=0 \Rightarrow d=0$$

$$b) \Rightarrow (d+4)=0 \Rightarrow d=-4$$

$$\Rightarrow (d-2)=0 \Rightarrow d=2$$

$$b)(d+4)(d-3)=0$$

$$\Rightarrow (d+4)=0 \Rightarrow d=-4$$

$$\Rightarrow (d-3)=0 \Rightarrow d=3$$

Since the factor (d+4) is located in both numerator and denominator it will cancel (divide out or reduce to one) so the values that make the numerator zero are 0 and 2 and the value that make expression undefined (denominator equal to 0) is 3

5. Solve the following equations:

$$\frac{9}{2x-3} = \frac{6}{3x-7} \Rightarrow LCD = (2x-3)(3x-7)$$

$$a) 9(3x-7) = 6(2x-3)$$

$$27x - 63 = 12x - 18$$

$$15x = 45 \Rightarrow x = 3$$

$$\frac{y+1}{3} - \frac{y+2}{6} = \frac{y+5}{4} \Rightarrow \frac{y+1}{3} - \frac{y+2}{2 \cdot 3} = \frac{y+5}{2^2} \Rightarrow LCD = 2^2 \cdot 3$$

$$(y+1) \cdot 2^2 - (y+2) \cdot 2 = (y+5) \cdot 3$$

$$b) 4y+4-2y-4=3y+15$$

$$2y = 3y+15$$

$$-y = 15 \Rightarrow y = -15$$

$$\frac{8}{x} + \frac{x+6}{3x} + \frac{x-4}{6x} = \frac{8}{9} \Rightarrow \frac{8}{x} + \frac{x+6}{3 \cdot x} + \frac{x-4}{2 \cdot 3 \cdot x} = \frac{8}{3^2} \Rightarrow LCD = 2 \cdot 3^2 \cdot x$$

$$8 \cdot 2 \cdot 3^2 + (x+6) \cdot 2 \cdot 3 + (x-4) \cdot 3 = 8 \cdot 2 \cdot x$$

$$c) 144 + 6x + 36 + 3x - 12 = 16x$$

$$9x + 168 = 16x$$

$$-7x = -168 \Rightarrow x = 24$$

d)

$$\frac{10}{x-3} + \frac{5}{x+1} = \frac{25}{x^2-2x-3} \Rightarrow \frac{10}{(x-3)} + \frac{5}{(x+1)} = \frac{25}{(x-3)(x+1)} \Rightarrow LCD = (x-3)(x+1) \frac{3}{x^2+2x-15} + \frac{4}{x^2-}$$

$$LCD = (x+3)(x-3)(x+5)$$

$$3 \cdot (x+3) + 4(x+5) = 8(x-3)$$

$$3x+9+4x+20 = 8x-24$$

$$7x+29 = 8x-24 \Rightarrow -x = -53 \Rightarrow x = 53$$

$$10(x+1) + 5(x-3) = 25$$

$$10x+10+5x-15 = 25$$

$$15x-5 = 25 \Rightarrow 15x = 30 \Rightarrow x = 2$$

$$\frac{3}{x^2+2x-15} + \frac{4}{x^2-9} = \frac{8}{x^2+8x+15} \Rightarrow \frac{3}{(x+5)(x-3)} + \frac{4}{(x+3)(x-3)} = \frac{8}{(x+5)(x+3)} \Rightarrow$$

$$LCD = (x+3)(x-3)(x+5)$$

e) $3 \cdot (x+3) + 4(x+5) = 8(x-3)$

$$3x+9+4x+20 = 8x-24$$

$$7x+29 = 8x-24 \Rightarrow -x = -53 \Rightarrow x = 53$$