Check Up Time...Am I now ready for this course??

1. Solve each of the following:

$(a)\left|5x+3\right|\le 12$

$(b)\ x^2-5x>14$

$x^2-5x-14>0$

$(x-7)(x+2)>0$

$x=7,-2$

$\{x<-2 \text{ or } x>7\}$

$(c)\ x^3+3x^2-24x=26$

$x^3+3x^2-24x-26=0$

$x=1$: $1+3-24-26\neq 0$

$x=-1$: $-1+3+24-26=0$

$\therefore (x+1)$ is a factor

$\pm 1, \pm 2, \pm 13, \pm 24$

Synthetic Division

-1	1	3	-24 $\quad -26$
		-1	-2 $\quad 26$
	1	2	-26 $\quad 0$ ← Remainder
	x^2	x	x^0

$(x+1)(x^2+2x-26)=0$

$x=\dfrac{-2\pm\sqrt{4-4(1)(-26)}}{2}$

$x=\dfrac{-2\pm\sqrt{108}}{2}$

$x=\dfrac{-2\pm 6\sqrt{3}}{2}$

$x=-1,\ -1\pm 3\sqrt{3}$

$(a)\left|5x+3\right| \le 12$

Case 1: BBP.

$5x+3 \ge 0$ then $5x+3 \le 12$

$5x \ge -3$ $\quad$ $5x \le 9$

$x \ge -\frac{3}{5}$ $\quad$ $x \le \frac{9}{5}$

$$-\frac{3}{5} \le x \le \frac{9}{5}$$

Case 2: BBN

$|-11| = 11$

$-(-11) = 11$

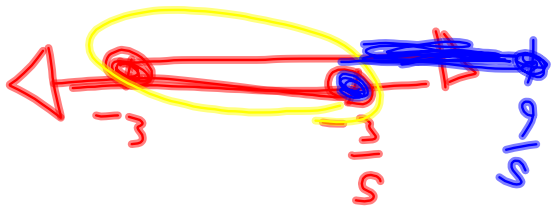
$5x+3 < 0$ then $-(5x+3) \le 12$

$5x < -3$ $\quad$ $-5x-3 \le 12$

$x < -\frac{3}{5}$ $\quad$ $-5x \le 15$

$x \ge -3$

$$-3 \le x < -\frac{3}{5}$$



Solution: $\left\{-3 \le x \le \frac{9}{5}\right\}$

2. Simplify each of the following:

(a) $\left(2-3\sqrt{8}\right)^2$

$= 4 - 12\sqrt{8} + 72$

$= 76 - 24\sqrt{2}$

(b) $\dfrac{2\sqrt{3}}{3-\sqrt{8}}\left(\dfrac{3+\sqrt{8}}{3+\sqrt{8}}\right)$ conjugate

$= \dfrac{6\sqrt{3}+2\sqrt{24}}{9-8}$

$= 6\sqrt{3}+4\sqrt{6}$

3. Evaluate the following:

$$27^{-\frac{2}{3}} + 3w^0 - \frac{2}{3^{-2}} - 3^{-2} + \frac{2}{5^{-2}}$$

$$= \frac{1}{\left(\sqrt[3]{27}\right)^2} + 3 - 2(3)^2 - \frac{1}{3^2} + 2(5)^2$$

$$= \frac{1}{9} + 3 - 18 - \frac{1}{9} + 50$$

$$= 35$$

Complex Numbers

Look at the following equation...

$x + 1 = 0 \ , x \in W$ ⟵ No Solution over the whole numbers

If we extend to the integers or real number systems then there will be a solution.

What about the equation $x^2 + 1 = 0 \ , x \in R$?

$$x^2 = -1$$

$$x = \sqrt{-1} \ ???$$

There is no solution over the real number system, therefore we extend into a new number system...the Complex Numbers.

⟶ $a + bi \ , a,b \in R$

Real Part — Imaginary Part

So what about this "*i*" that appears?

$i^2 = -1$

Most Important prinicple in complex number system ⟶ $i = \sqrt{-1}$

What is $\sqrt{-36}$? $x \in C$

$\sqrt{36(-1)} \Rightarrow \boxed{i^2 = -1}$

$\sqrt{36}\, i^2$

$= \pm 6i$

Basic Operations involving Complex Numbers

I. Addition and Subtraction

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

Collect "real" terms — Collect "imaginary" terms

$(2-7i)-(4+i)$
$=-2-8i$

Example:
Express the following complex expression in standard form:

$$2(3-5i)-(7-5i)+2(-1+i)$$ → a+bi

$6-10i-7+5i-2+2i$
$=-3-3i$

II. Multiplication and Powers

Examples:

a) $(2-i)(-1+3i)$

$=-2+6i+i(-3i^2)$ ×3
$=1+7i$

b) $3-2(5-2i)^2$

c) $2i^5-i^8+(2i^3)^5$

III. Division

Before we can divide we must first review the concept of conjugates...

Conjugates

$$a+bi \Leftrightarrow a-bi$$

Means to take conjugate

Examine what happens when you multiply complex conjugates...

$$(2-5i)(2+5i)$$

Now we are ready to try division...

Multiply the numerator and denominator by the conjugate of the denominator

Example:

a) $\dfrac{2+4i}{1-i}$

b) $\dfrac{(2-i)(-1+3i)}{(-3+2i)^2}$