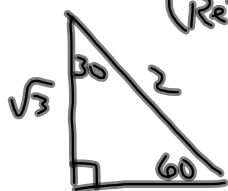# Warm Up...

1. Express $1 - \sqrt{3}i$ in polar form.

$(1, -\sqrt{3}) \Rightarrow Q4$

$\tan\theta = \sqrt{3}$
(Ref $\angle = 60°$)
$\theta = 300°$

$r = \sqrt{1+3}$
$r = 2$

$= 2(\cos 300° + i\sin 300°)$

$= 2 \text{ cis } 300°$

2. Express $-3 - 4i$ in polar form.

$r = 5$ Quad. 3: $\theta = 233°$ $= 5 \text{ cis } 233°$

3. Express the number 4 in polar form.

$4 + 0i$
$(4, 0)$
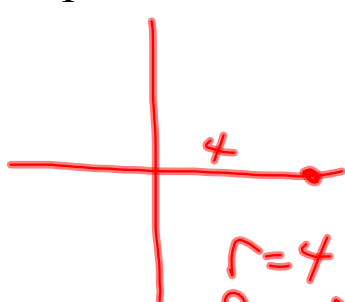
$r = 4$
$\theta = 0°$ $\Big\}$ $= 4 \text{ cis } 0°$
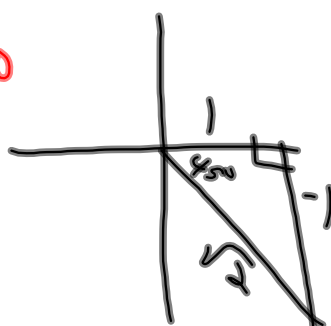
4. Express the polar form $\sqrt{8}(\cos 1395° + i\sin 1395°)$
as a complex number in rectangular form.

$1395° \div 360$

$= 3.875$

$= 0.875 \times 360°$

$= 315°$

$= \sqrt{8}\left(\frac{1}{\sqrt{2}} + i\left(\frac{-1}{\sqrt{2}}\right)\right)$

$= \sqrt{4} - \sqrt{4}i$

$= 2 - 2i$

Product and Quotient of Complex Numbers in Polar Form

Let $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ ...

Now let's examine $z_1 \bullet z_2$ and $\dfrac{z_1}{z_2}$

$$z_1 \bullet z_2$$

$$\left[r_1(\cos \theta_1 + i \sin \theta_1) \right] \times \left[r_2(\cos \theta_2 + i \sin \theta_2) \right]$$

$$r_1 r_2 \left[(\cos \theta_1 + i \sin \theta_1)(\cos \theta_2 + i \sin \theta_2) \right]$$

$$r_1 r_2 \left[\cos \theta_1 \cos \theta_2 + i \cos \theta_1 \sin \theta_2 + i \sin \theta_1 \cos \theta_2 + i^2 \sin \theta_1 \sin \theta_2 \right]$$

$$r_1 r_2 \left[\left(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \right) + i \left(\cos \theta_1 \sin \theta_2 + \sin \theta_1 \cos \theta_2 \right) \right]$$

$$r_1 r_2 \left[\cos (\theta_1 + \theta_2) + i \sin (\theta_1 + \theta_2) \right]$$

CONCLUSIONS...

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ ...

$$z_1 \bullet z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

- multiply all "r" values together
- add all angles together

&

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

- divide "r" values
- subtract angles

Examples:

Evaluate:

$a + bi$

$$\sqrt{2}\left(\cos 57^\circ + i\sin 57^\circ\right) \bullet 2\sqrt{6}\left(\cos 213^\circ + i\sin 213^\circ\right)$$

$= \left(\sqrt{2}\right)\left(2\sqrt{6}\right) cis \left(57^\circ + 213^\circ\right)$

$= 4\sqrt{3} \; cis \; 270^\circ$

$= 4\sqrt{3} \left(0 + -1i\right) = \boxed{-4\sqrt{3}i} \quad + (0,y)$

Evaluate: $\dfrac{2\sqrt{2}\left(\cos 135^\circ + i\sin 135^\circ\right)}{6\left(\cos 300^\circ + i\sin 300^\circ\right)}$

$= \dfrac{\sqrt{2}}{3} \; cis \left(135^\circ - 300^\circ\right)$

$= \dfrac{\sqrt{2}}{3} \; cis\left(-165^\circ\right)$

$= \dfrac{\sqrt{2}}{3} \left(-0.986 - 0.259i\right)$

$= 0.46 - 0.12i$

EXERCISE: Express the following in the form "a + b*i*"...

```
((4-4i√(3))(2√(3
)+2i)(1+i))/((5-
5i)(-√(3)+i))
            -3.2i
Ans▶Frac
            -16/5i
■
```

$$\frac{\left(4 - 4i\sqrt{3}\right)\left(2\sqrt{3}+2i\right)\left(1+i\right)}{\left(5-5i\right)\left(-\sqrt{3}+i\right)}$$

Now - let's switch into **polar form** to solve the problem!!!

$(4, -4\sqrt{3})$, Q4

$r = \sqrt{16+48}$

$r = 8$

$\tan\theta = \frac{4\sqrt{3}}{4} = \sqrt{3}$

Ref $\angle = 60°$

$\theta = 300°$

$= 8\text{cis}\,300°$

$(2\sqrt{3}, 2)$, Q1

$r = \sqrt{12+4}$

$r = 4$

$\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

Ref $\angle = 30°$

$= 4\text{cis}\,30°$

$5 - 5i \Rightarrow (5, -5)$
Q4

$r = \sqrt{50} = 5\sqrt{2}$

$\tan\theta = 1$

Ref $\angle = 45$, Q4

$\theta = 315°$

$= 5\sqrt{2}\text{cis}\,315°$

$(-\sqrt{3}, 1)$ Q2

$r = 2$

$\tan\theta = \frac{1}{\sqrt{3}}$

Ref $\angle = 30$, Q2

$\theta = 150°$

$= 2\text{cis}\,150°$

$= 1 + i \Rightarrow (1, 1)$ Q1
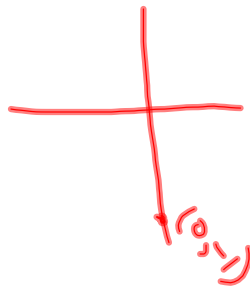
$= r = \sqrt{2}$

$\tan\theta = 1$

Ref $\angle = 45°$

$= \sqrt{2}\,\text{cis}\,45°$

$= \frac{\left(8\text{cis}\,300\right)\left(4\text{cis}\,30°\right)\left(\sqrt{2}\,\text{cis}\,45°\right)}{\left(5\sqrt{2}\text{cis}\,315°\right)\left(2\text{cis}\,150°\right)}$

$= \frac{32\sqrt{2}\,\text{cis}\,375°}{10\sqrt{2}\,\text{cis}\,465°}$

$= \frac{16}{5}\text{cis}\,(-90°)$
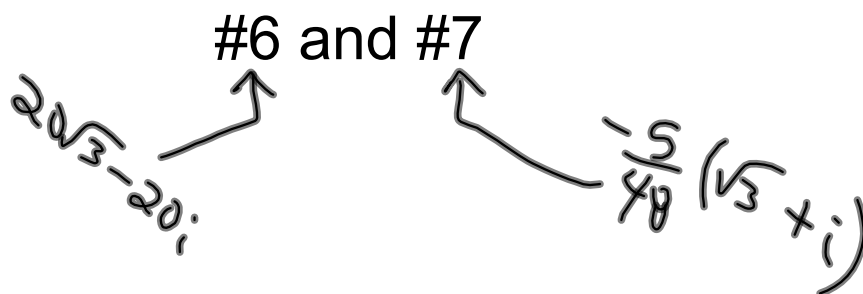
$= \frac{16}{5}(0 + -1i)$

$= \frac{-16}{5}i$

Homework...

Worksheet - Polar Form

We just did #5 together!!!

#6 and #7

$2\sqrt{3} - 2i$

$-\dfrac{5}{40}(\sqrt{3} + i)$

Attachments

Worksheet - DeMoivres Theorem.doc