Warm-Up

#7

Evaluate: $\dfrac{(1-i)(5+5i)}{(3-3i\sqrt{3})(-4\sqrt{3}-4i)}$

$(1,-1)$, Q4

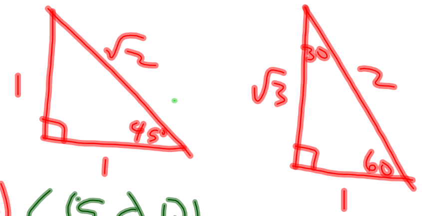
$r=\sqrt{2}$

$\tan\theta = 1$

(Ref∡ = 45°, Q4)

$\theta = 315°$

$\Rightarrow \underline{\sqrt{2}\,\text{cis}\,315°}$



$(5,5)$ Q1

$r=\sqrt{25+25}$

$r=5\sqrt{2}$

$\tan\theta = \frac{5}{5} = 1$

(Ref∡ = 45°, Q1)

$\Rightarrow \underline{5\sqrt{2}\,\text{cis}\,45°}$

$(3,-3\sqrt{3})$ Q4

$r=\sqrt{9+27}$

$r=6$

$\tan\theta = \frac{3\sqrt{3}}{3}$

$=\sqrt{3}$

(Ref∡ = 60°, Q4)

$\theta = 300°$

$\Rightarrow \underline{6\,\text{cis}\,300°}$

$(-4\sqrt{3},-4)$ Q3

$r=\sqrt{48+16}$

$r=8$

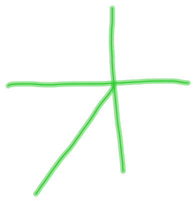
$\tan\theta = \frac{4}{4\sqrt{3}}$

$=\frac{1}{\sqrt{3}}$

(Ref∡ = 30°, Q3)

$\theta = 210°$

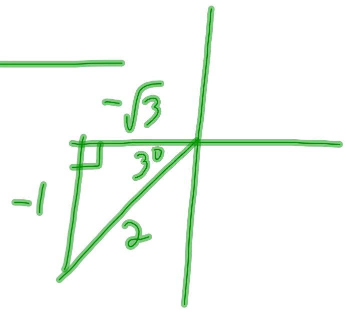
$\Rightarrow \underline{8\,\text{cis}\,210°}$



$$=\frac{(\sqrt{2}\,\text{cis}\,315°)(5\sqrt{2}\,\text{cis}\,45°)}{(6\,\text{cis}\,300°)(8\,\text{cis}\,210°)}$$

$$=\frac{10\,\text{cis}\,360°}{48\,\text{cis}\,510°}$$

$$=\frac{5}{24}\,\text{cis}\,(-150°)$$



$$=\frac{5}{24}\left(\cos(-150)+i\sin(-150)\right)$$

$$=\frac{5}{24}\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)$$

What about something like this???

Evaluate the following:

Huh???

$$(-1+i)^{10}$$

$$\underbrace{(-1+i)(-1+i)\ldots(-1+i)}_{10 \text{ of these}}$$

$(-1,1)$, Q2

$r=\sqrt{2}$

$\tan\theta = 1$

(Ref $\measuredangle = 45°$; Q2)

$\theta = 135°$

$$\underbrace{(\sqrt{2}\,\text{cis}\,135°)(\sqrt{2}\,\text{cis}\,135°)\ldots(\sqrt{2}\,\text{cis}\,135°)}_{10 \text{ of these}}$$

$$=(\sqrt{2})^{10}\,\text{cis}(10\times 135°)$$

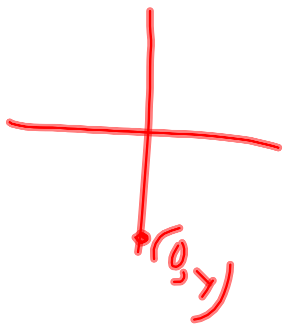
$$=32\,\text{cis}(1350°)$$

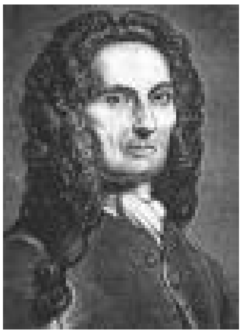
$$=32(0+-1i)$$

$$=-32i$$

Principle Angle

270°





THEOREM!!!

Demoivre's

$$\boxed{[r(\cos\theta + i\sin\theta)]^n = r^n[\cos(n\theta) + i\sin(n\theta)]}$$

$$(r\,cis\,\theta)^n = r^n cis(n\theta)$$

Example: Simplify the following using the polar form of a complex number.

$$(1-i)^6$$

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(1-i)^6
                8i
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$(1,-1)$ Q4

$r = \sqrt{2}$

$\tan\theta = 1$

(Ref ∡ = 45°, Q4)

$\theta = 315°$

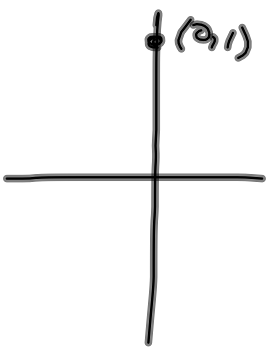
$$\Longrightarrow \left(\sqrt{2}\, cis\, 315°\right)^6$$

$$= \left(2^{\frac{1}{2}}\right)^6 cis\,(315 \times 6)$$

$$= 8\, cis\, 1890°$$

$$= 8(0 + 1i)$$

$$= \underline{8i}$$



Example:

SOLUTION: $-24 - 24i$

$$\frac{\left(-1+i\sqrt{3}\right)^9 (4i)^8 (3+3i)}{\left(-4\sqrt{3}-4i\right)^6 (1-i)^8}$$

$(-1,\sqrt{3})$, Q2

$r = 2$

$\tan\theta = \sqrt{3}$

(Ref∡ $= 60°$, Q2)

$\theta = 120°$

$= \left(2\,\text{cis}\,120°\right)^9$

$= 2^9\,\text{cis}\,1080°$

$4i \Rightarrow (0,4)$

$r = 4 \quad \theta = 90°$

$\Rightarrow \left(2^2\,\text{cis}\,90°\right)^8$

$\Rightarrow 2^{16}\,\text{cis}\,720°$

$(3,3)$, Q1

$r = \sqrt{9+9}$

$r = 3\sqrt{2}$

$\tan\theta = 1$

(Ref∡ $45°$, Q1)

$\theta = 45°$

$\Rightarrow 3\sqrt{2}\,\text{cis}\,45°$

$(-4\sqrt{3}, -4)$ Q3

$r = \sqrt{48+16}$

$r = 8$

$\tan\theta = \frac{1}{\sqrt{3}}$

(Ref∡ $= 30°$, Q3)

$\theta = 210°$

$= \left(2^3\,\text{cis}\,210°\right)^6$

$= 2^{18}\,\text{cis}\,1260°$

$(1,-1)$ Q4

$r = \sqrt{2}$

$\tan\theta = 1$

(Ref∡ $= 45°$, Q4)

$\theta = 315°$

$= \left(2^{1/2}\,\text{cis}\,315°\right)^8$

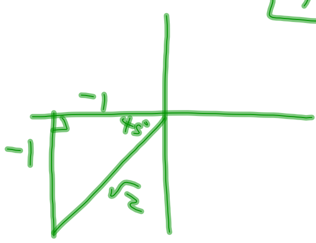
$\Rightarrow 2^4\,\text{cis}\,2520°$

$$= \frac{\left(2^9\,\text{cis}\,1080°\right)\left(2^{16}\,\text{cis}\,720°\right)\left(3\sqrt{2}\,\text{cis}\,45°\right)}{\left(2^{18}\,\text{cis}\,1260°\right)\left(2^4\,\text{cis}\,2520°\right)}$$

$$= \left(\frac{2^{25}\cdot 3\sqrt{2}}{2^{22}}\right)\text{cis}\left(1080+720+45-1260-2520\right)$$

$$= 24\sqrt{2}\,\text{cis}(-1935°)$$

P.Angle: $-135°$
$+360°$
$225°$



$$= 24\sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$$

$$= -24 - 24i$$

BONUS

Evaluate:

$$\frac{\left(-\frac{3}{2}+\frac{3i\sqrt{3}}{2}\right)^{4}\left(2\sqrt{3}+2i\right)^{3}}{\left(9\sqrt{2}+9i\sqrt{2}\right)^{2}}$$