# Evaluating Limits

## I. Using a Graph:

- We looked at this in the previous two examples

## II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x\to -2}\frac{x^2-2x+1}{x+3} \qquad \lim_{x\to 3}\left(16-x^2\right)$$

- Indeterminate limits... $\Rightarrow$ Direct substitution leads to $\frac{0}{0}$
  - $\Rightarrow$ Factor
  - $\Rightarrow$ Rationalize
  - $\Rightarrow$ Expand
  - $\Rightarrow$ Find Common Denominators

Examples:

$$\lim_{x\to 3}\frac{x^2-6x+9}{9-x^2} = \frac{0}{0}$$

$$\lim_{x\to 3}\frac{-1(x-3)(x-3)}{(3-x)(3+x)}$$

$$=\frac{(-1)(3-3)}{3+3}$$

$$=\frac{0}{6}$$

$$=0$$

Rationalize

$$\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}\cdot\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$$

$$\lim_{h\to 0}\frac{(4+h)-4}{(h)(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0}\frac{h\,1}{h(\sqrt{4+h}+2)}$$

$$=\frac{1}{\sqrt{4+0}+2}$$

$$=\frac{1}{4}$$

Try these...remember to use your algebra skills
to try and eliminate the **indeterminate form**.

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)^2-(x-2)^2} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{x(x+3)}{(x^2+4x+4)-(x^2-4x+4)}$$

$$\lim_{x\to 0}\frac{x(x+3)}{8x}$$

$$= \frac{3}{8}$$

diff. of squares

$$\big((x+2)-(x-2)\big)\big((x+2)+(x-2)\big)$$

$$(4)(2x)$$

$$\lim_{x\to -2}\frac{x^4-16}{x^3+8}$$

$$\lim_{x\to -2}\frac{(x^2-4)(x^2+4)}{(x+2)(x^2-2x+4)}$$

$$\lim_{x\to -2}\frac{(x-2)(x+2)(x^2+4)}{(x+2)(x^2-2x+4)}$$

$$= \frac{(-4)(8)}{4+4+4} = -\frac{8}{3}$$

$$\lim_{x\to 2}\frac{(x+2)^2-16}{x^2-4}$$

$x^2+4x+4-16$

$x^2+4x-12$

$(x+6)(x-2)$

$$\lim_{x\to 2}\frac{(x+2-4)(x+2+4)}{(x-2)(x+2)}$$

$$\lim_{x\to 2}\frac{(x-2)(x+6)}{(x-2)(x+2)}$$

$$= \frac{8}{4} = 2$$

$$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2} \div$$

$$\lim_{x\to 2}\left(\frac{2-x}{2x}\right)\cdot\frac{1}{x-2}$$

$$= -\frac{1}{4}$$

Be careful...

$$\frac{2+x}{x+2} + 1$$

# Homework...

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