$$\frac{3}{w^2} - \frac{2}{7}$$

$$\frac{3(7) - 2w^2}{7w^2}$$

$\times 7$ $\times w^2$

$x-7$

$(\sqrt{x}-\sqrt{7})(\sqrt{x}+\sqrt{7})$

x^2-49

$(x-7)(x+7)$

x^3-8

$x-7$

$(\sqrt[3]{x}-\sqrt[3]{7})\left((\sqrt[3]{x})^2+\sqrt[3]{7x}+(\sqrt[3]{7})^2\right)$

Warm Up

Evaluate the following limits, if they exist:

1. $\lim\limits_{x\to 8} \dfrac{x-8}{\sqrt[3]{x}-2}$

$$\lim_{x\to 8} \frac{\cancel{(\sqrt[3]{x}-2)}\left((\sqrt[3]{x})^2+2\sqrt[3]{x}+4\right)}{\cancel{\sqrt[3]{x}-2}}$$

$$= (\sqrt[3]{8})^2 + 2\sqrt[3]{8} + 4$$

$$= 4+4+4 = 12$$

2. $\lim\limits_{x\to 3} \dfrac{x^{-2}-3^{-2}}{x-3}$

$$\lim_{x\to 3} \frac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$$

$$\lim_{x\to 3} \left(\frac{9-x^2}{9x^2}\right)\cdot\frac{1}{x-3}$$

$$\lim_{x\to 3} \frac{-1\cancel{(3-x)}(3+x)}{9x^2}\cdot\frac{1}{\cancel{x-3}}$$

$$= \frac{-1(6)}{9(9)} = \frac{-6}{81} = \frac{-2}{27}$$

3. $\lim\limits_{x\to 1} \dfrac{x^3-1}{x^3-x^2-4x+4}$

$\underline{x=1}$

$1-1-4+4=0$

$\therefore$ $(x-1)$ is a factor

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -4 & 4 \\ & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$(x-1)(x^2-4)$

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x^2-4)}$$

$$= \frac{1+1+1}{1-4}$$

$$= \frac{3}{-3} = -1$$

4. $\lim\limits_{h\to 0} \dfrac{(a+h)^2-a^2}{h}$

$$\lim_{h\to 0} \frac{[(a+h)-a][(a+h)+a]}{h}$$

$$\lim_{h\to 0} \frac{\cancel{(h)}(2a+h)}{\cancel{h}}$$

$$= 2a+0$$

$$= 2a$$

Recall from our prior discussions that ...

1 Theorem $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$

Let's look at a couple of unique functions:

1) $\lim_{x \to 1} \sqrt{x-1} = \text{DNE}$

2) $\lim_{x \to -2} \frac{|x+2|}{x+2}$

even indexed radical by itself

$\lim_{x \to 1^-} \sqrt{0.9999 - 1}$

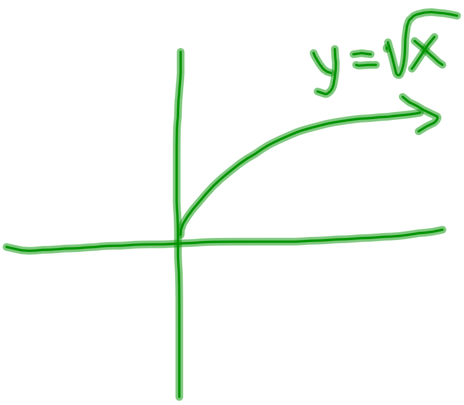
$\lim_{x \to 1^-} \sqrt{-0.000..1}$

DNE

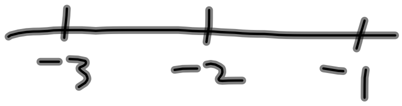
$\lim_{x \to 1^+} \sqrt{1.0000.1 - 1}$

$= \sqrt{0.0000..1}$

$= 0$



2) $\lim_{x \to -2} \frac{|x+2|}{x+2}$ = DNE



$$\lim_{x \to -2^-} \frac{|-2.000..1+2|}{-2.000..1+2}$$

$$= \frac{|\text{small } (-)\ \#|}{\text{same small } (-)\ \#}$$

$$= \frac{\text{small } (+)\ \#}{\text{same small } (-)\ \#}$$

$$= -1$$

$$\lim_{x \to -2^+} \frac{|-1.999...+2|}{-1.999...+2}$$

$$= \frac{\cancel{\text{small } (+)\ \#}}{\cancel{\text{same small}}\ (+)\ \#}$$

$$= 1$$

Piecewise Defined Functions

Definition:
- **Functions defined by different formulas in different parts of their domains**

Example:

$$f(x)=\begin{cases} x+3 & if\ x\le 2 \\ x^2-2 & if\ x>2 \end{cases}$$

1) Determine *f(1)* , *f(3)*, and *f(2)*.
2) Sketch *f(x)*.

$1<2$

$f(1)=1+3=4$

$f(3)=(3)^2-2=7$

$f(2)=2+3=5$

① $y=x+3$

x	y
2	5
0	3

② $y=x^2-2$

x	y
2	2
3	7

parabola
$V(0,-2)$