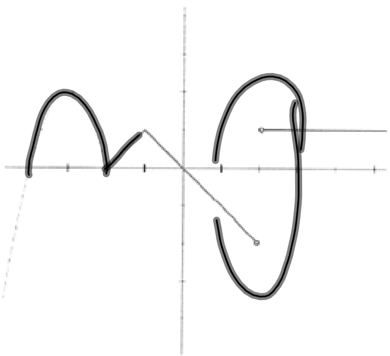
WARM-UP: LIMITS OCT. 2010

1. Use the graph of $f(x)$ shown to evaluate the following: [4]



$\lim_{x\to -2^+} f(x) =$ 0

$\lim_{x\to 2^-} f(x) =$ -2

$\lim_{x\to 2} f(x) =$ DNE

$f(2) =$ DNE

2. Evaluate each of the following limits. If a limit does not exist provide a reason. [16]

(a) $\lim_{x\to 0}\frac{2-\sqrt{4-x}}{x}\left(\frac{2+\sqrt{4-x}}{2+\sqrt{4-x}}\right)$

$$\lim_{x\to 0}\frac{\cancel{4}-\cancel{(4}-x)}{\cancel{x}(2+\sqrt{4-x})}$$

$$=\frac{1}{2+\sqrt{4}}$$

$$=\frac{1}{4}$$

(b) $\lim_{w\to 5}\frac{\frac{1}{w}-\frac{1}{5}}{25-w^2}$

$$\lim_{w\to 5}\left(\frac{\cancel{5-w}}{5w}\right)\cdot\frac{1}{\cancel{(5-w)}(5+w)}$$

$$=\frac{1}{25(10)}=\frac{1}{250}$$

(c) $\lim_{x\to a}\frac{(x+a)^2-4a^2}{x-a}$

$$\lim_{x\to a}\frac{\left[\cancel{(x+a)-2a}\right]\left[(x+a)+2a\right]}{\cancel{x-a}}\quad \cancel{x-a}$$

$$=(a+a)+2a$$

$$=4a$$

(d) $\lim_{x\to -3}\frac{x^3+27}{x^2+x-6}$

$$\lim_{x\to -3}\frac{\cancel{(x+3)}(x^2-3x+9)}{\cancel{(x+3)}(x-2)}$$

$$=\frac{9+9+9}{-5}$$

$$=\frac{27}{-5}$$

Piecewise Defined Functions

Definition:

- **Functions defined by different formulas in different parts of their domains**

Example:

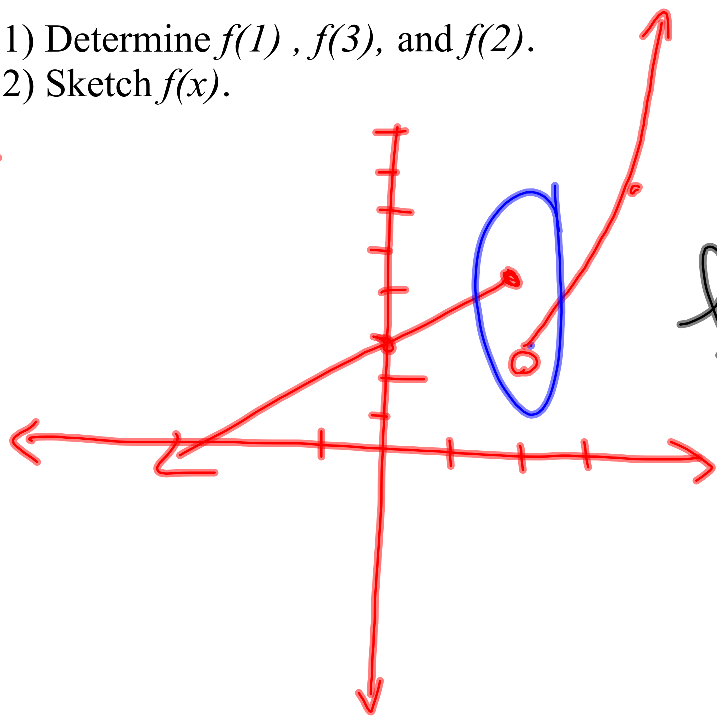
$$f(x)=\begin{cases} x+3 & if\ x\le 2 \\ x^2-2 & if\ x>2 \end{cases}$$

v(0,-2)

1) Determine *f(1)* , *f(3),* and *f(2)*.
2) Sketch *f(x)*.

x	y
2	2
3	7

x	y
2	5
0	3



$$\lim_{x\to 2^-} f(x) \ne \lim_{x\to 2^+} f(x)$$

$$f(x) = \begin{cases} x+3 & if\ x \le 2 \\ x^2 - 2 & if\ x > 2 \end{cases}$$

greater than 2

$\lim_{x \to 2} f(x) = ?$

less than 2 $\lim_{x \to 2^-} f(x)$

$= 2+3$

$= 5$

$\lim_{x \to 2^+} f(x)$

$= (2)^2 - 2$

$= 4-2$

$= 2$

$\lim_{x \to 2} f(x) = $ D.N.E.

Sketch the following piecewise function:

$$f(x)=\begin{cases} 1\quad \frac{1}{2}x-2 & \text{if } x<-2 \\ 2\quad -1 & \text{if } -2\leq x\leq 1 \\ 3\quad (x-2)^2+1 & \text{if } x \geq 1 \end{cases}$$

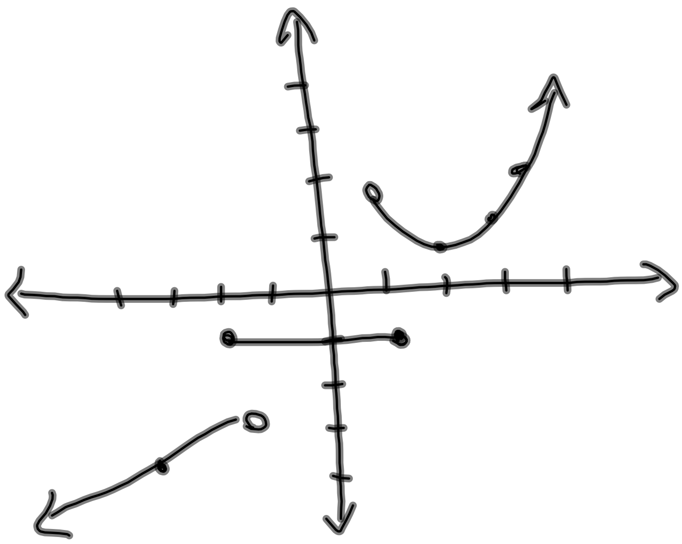
1

x	y
-2	-3
-4	-4

2 $y=-1$

3 $V(2,1)$

x	y
1	2



Absolute value function...the hidden piecewise function!

What about this function? → $f(x) = |x|$

$|x| = 6$

$x = 6$ OR -6

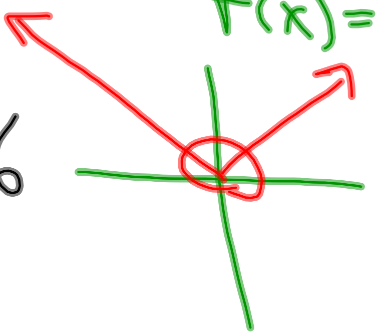
$|6| = 6$

$|-6| = 6$

$-(-6) = 6$

Write <u>without</u> absolute Bars

$$f(x) = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$



More Practice...

- Express the following absolute value function as a piecewise function
- Sketch the function

$$f(x) = |x-3|$$

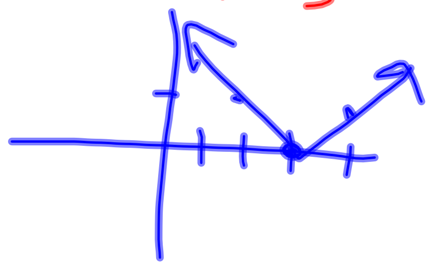
BBP

$V(3,0)$

$x-3 \geq 0$

$x \geq 3$

$$f(x) = \begin{cases} x-3, & x \geq 3 \\ -x+3, & x < 3 \end{cases}$$



Sketch:

$y = a(x-h)^2 + q$

$V(h, q)$

$y = a|x-p| + q$

$V(p, q)$