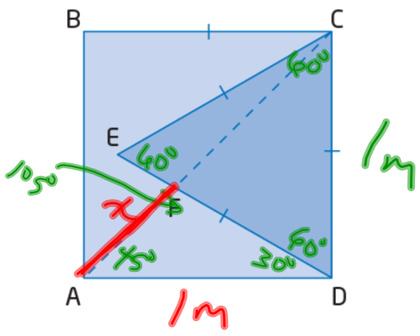
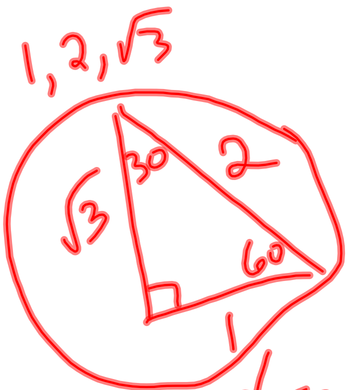
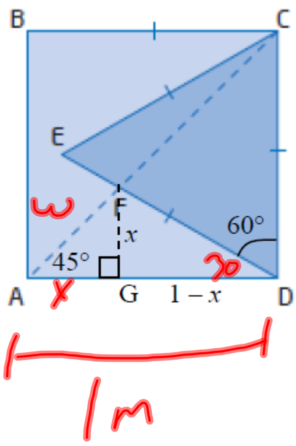
Let's revisit the bonus problem...

A square, ABCD, has a perimeter of 4 m. △CDE is an equilateral triangle inside the square. The intersection of AC and DE occurs at point F. What is the exact length of AF?







$$\frac{x}{\sin 30°} = \frac{1}{\sin 105°}$$

$$x = \frac{\sin 30°}{\sin 105°}$$

$$\tan 30° = \frac{x}{1-x}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{1-x}$$

$$1 - x = \sqrt{3}x$$

$$1 = \sqrt{3}x + x$$

$$1 = (\sqrt{3}+1)x$$

$$x = \frac{1}{\sqrt{3}+1}$$

$$\left(\frac{1}{\sqrt{3}+1}\right)^2 + \left(\frac{1}{\sqrt{3}+1}\right)^2 = \omega^2$$

⋮

Dividing Radical Expressions

Property:

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \text{ if } b \neq 0$$

Let's look at a few examples...

$$\frac{\sqrt{15}}{\sqrt{3}} = \sqrt{\frac{15}{3}} = \sqrt{5}$$

$$\frac{24\sqrt{48}}{3\sqrt{12}} = \frac{24(4\sqrt{3})}{6\sqrt{3}} = \frac{96\sqrt{3}}{6\sqrt{3}} = 16(1) = 16$$

$$= 8\sqrt{4}$$

$$= 8(2) = 16$$

$$\frac{7\sqrt[3]{320}}{14\sqrt[3]{10}}$$

$$= \frac{1}{2}\sqrt[3]{32}$$

$$= \frac{1}{2}(2\sqrt[3]{4})$$

$$= \sqrt[3]{4}$$

$$\frac{-8x^5\sqrt{48x^{11}}}{3x^2\sqrt{2x^3}}$$

$$= -\frac{8}{3}x^3\sqrt{24x^8} \rightarrow -\frac{8}{3}x^3(2x^4\sqrt{6})$$

$$= -\frac{16}{3}x^7\sqrt{6}$$

What if the everything does not divide evenly??

$$\frac{\sqrt{10}}{\sqrt{6}} = ??$$

$$\frac{3}{4}\left(\frac{7}{7}\right) = \frac{21}{28}$$

Rationalizing the Denominator

Rationalize

rationalize

- convert to a rational number without changing the value of the expression
- If the radical is in the denominator, both the numerator and denominator must be multiplied by a quantity that will produce a rational denominator.

$$\frac{\sqrt{10}}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right) = \frac{\sqrt{60}}{6} = \frac{2\sqrt{15}}{6} = \frac{\sqrt{15}}{3}$$

$$\frac{\sqrt{5}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{15}}{3}$$

$$\frac{6}{\sqrt{12}}\left(\frac{\sqrt{12}}{\sqrt{12}}\right)$$

$$= \frac{6\sqrt{12}}{12}$$

$$= \frac{\cancel{12}\sqrt{3}}{\cancel{12}}$$

$$= \sqrt{3}$$

$$\frac{\sqrt{28}}{\sqrt{8}}\left(\frac{\sqrt{8}}{\sqrt{8}}\right) = \frac{\sqrt{224}}{8}$$

$$= \frac{4\sqrt{14}}{8}$$

$$= \frac{\sqrt{14}}{2}$$

OR

$$\frac{\cancel{2}\sqrt{7}}{\cancel{2}\sqrt{2}}$$

$$\frac{\sqrt{7}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{14}}{2}$$

$$\frac{12\sqrt{18}}{8\sqrt{8}}\left(\frac{\sqrt{8}}{\sqrt{8}}\right)$$

$$\frac{12(3\sqrt{2})}{8(2\sqrt{2})}$$

$$= \frac{36\cancel{\sqrt{2}}}{16\cancel{\sqrt{2}}}$$

$$= \frac{9}{4}$$

$$\frac{12\sqrt{144}}{8(8)}$$

$$= \frac{12(12)}{64}$$

$$= \frac{144}{64}$$

$$= \frac{9}{4}$$

$$= \frac{6\sqrt{9}}{4\sqrt{4}}$$

$$= \frac{6(3)}{4(2)}$$

$$= \frac{18}{8}$$

$$= \frac{9}{4}$$

$$\frac{-3\sqrt{8a}}{a\sqrt{5a^3}}\left(\frac{\sqrt{5a^3}}{\sqrt{5a^3}}\right)$$

$$= \frac{-3\sqrt{40a^4}}{5a^4}$$

$$= \frac{-6a^2\sqrt{10}}{5a^4}$$

$$6^{-1} = \frac{1}{6}$$

$$= \frac{-6\sqrt{10}}{5a^2}$$

$$(-6)^{-1}$$

OR

$$= -\frac{6}{5}a^{-2}\sqrt{10}$$

What about rationalizing with other indicies??

$$\frac{9\sqrt{24}}{\sqrt[3]{6}}\left(\frac{(\sqrt[3]{6})^2}{(\sqrt[3]{6})^2}\right)$$

$$(\sqrt[3]{6})(\sqrt[3]{6})(\sqrt[3]{6})$$

$$6^{1/3}\cdot 6^{1/3}\cdot 6^{1/3}$$

$$6^{3/3}$$

$$6^1$$

$$=\frac{9\sqrt{24}(\sqrt[3]{6})^2}{6}$$

$$=\frac{18\sqrt{6}(\sqrt[3]{6})^2}{6}$$

$$=3\sqrt{6}(\sqrt[3]{6})^2$$

$$\frac{5\sqrt[3]{4w^2}}{w\sqrt[5]{8}}\left(\frac{(\sqrt[5]{8})^4}{(\sqrt[5]{8})^4}\right)$$

Ex. 5.2

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Practice Problems...(monomial denominators)

#6, 7, 8, 15, 20