UNDEFINED VALUES For what values of the variable is the rational expression undefined?

SIMPLIFYING EXPRESSIONS Simplify the expression if possible.

$$\frac{-18x^2}{12x}$$

$$= -\frac{3x}{2}, x \neq 0$$

$$5x - 12 = 0$$
$$5x = 12$$
$$\boxed{x = \frac{12}{5}}$$

$$\frac{12 - 5x}{10x^2 - 24x}$$

$$= \frac{-1}{12 - 5x} \over 2x(5x - 12)$$

$$= \frac{-1}{2x}, x \neq 0, \frac{12}{5}$$

$$\frac{x^3 - x}{x^3 + 5x^2 - 6x}$$

$$\frac{x(x^2 - 1)}{x(x^2 + 5x - 6)}$$

$$\frac{x(x-1)(x+1)}{x(x+6)(x-1)}$$

$$\frac{x+1}{x+6}, x \neq -6, 0, 1$$

$$\frac{5 - x}{x^2 - 8x + 15}$$

$$\frac{-1}{5 - x} \over (x-3)(x-5)$$

$$= \frac{-1}{x-3}, x \neq 3, 5$$

$$\frac{x^3 + 9x^2 + 14x}{x^2 - 4}$$

$$\frac{x(x^2 + 9x + 14)}{(x-2)(x+2)}$$

$$\frac{x(x+7)(x+2)}{(x-2)(x+2)}$$

$$= \frac{x(x+7)}{x-2}, x \neq \pm 2$$

$$\frac{2x^2 + 11x - 6}{x + 6} \quad -12$$

$$2x^2 + 12x - 1x - 6$$
$$2x(x+6) - 1(x+6)$$
$$\frac{(x+6)(2x-1)}{x+6}$$

$$= 2x - 1, x \neq 6$$

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8. d) $\dfrac{10k^2 + 55k + 75}{20k^2 - 10k - 150}$

$$\dfrac{5(\overset{\overset{\displaystyle 30}{\frown}}{2k^2 + 11k + 15})}{10(\underset{\underset{\displaystyle -30}{\frown}}{2k^2 - k - 15})}$$

$2k^2 + 5k + 6k + 15$
$k(2k+5) + 3(2k+5)$
$(2k+5)(k+3)$

$2k^2 - 6k + 5k - 15$
$2k(k-3) + 5(k-3)$
$(k-3)(2k+5)$

$$\dfrac{\cancel{5}^{1}\, (\cancel{2k+5})(k+3)}{\cancel{10}_{2}(k-3)(\cancel{2k+5})}$$

$$\dfrac{k+3}{2(k-3)}, \quad k \neq 3, -\dfrac{5}{2}$$

8. f) $\dfrac{5(x^2-y^2)}{x^2-2xy+y^2}$

$\dfrac{5(x-y)(x+y)}{(x-y)(x-y)}$

$= \dfrac{5(x+y)}{x-y}, \; x \neq y$

26. d) $\dfrac{(x^2+4x+4)^2-10(x^2+4x+4)+9}{(2x+1)^2-(x+2)^2}$

$$\begin{cases} \text{☺}^2-10\,\text{☺}+9 \\ (\text{☺}-9)(\text{☺}-1) \end{cases}$$

Let $m = x^2+4x+4$

Now.. $m^2-10m+9$

Re-Sub. $(m-9)(m-1)$
$(x^2+4x+4-9)(x^2+4x+4-1)$
$(x^2+4x-5)(x^2+4x+3)$
$(x-1)(x+5)(x+3)(x+1)$ ← — Numerator

$\sqrt{(2x+1)^2}-\sqrt{(x+2)^2} \implies \hat{\wedge}^2 - \text{✳}^2$

$(\hat{\wedge}-\text{✳})(\hat{\wedge}+\text{✳})$

$[(2x+1)-(x+2)][(2x+1)+(x+2)]$
$(x-1)(3x+3)$
$3(x-1)(x+1)$ ← — Denominator

$\dfrac{\cancel{(x-1)}(x+5)(x+3)\cancel{(x+1)}}{3\cancel{(x-1)}\cancel{(x+1)}}$

$= \dfrac{1}{3}(x+5)(x+3) \,, \; x \neq \pm 1$

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Multiplying and Dividing Rational Expressions

Focus on...

- comparing operations on rational expressions to the same operations on rational numbers
- identifying non-permissible values when performing operations on rational expressions
- determining the product or quotient of rational expressions in simplest form

> **Objective: Multiply and divide rational expressions.**
>
> Multiplying and dividing rational expressions is very similar to the process we use to multiply and divide fractions.

Let's recall how to multiply and divide rational numbers...

Example

Calculate $\dfrac{2}{3} \times \dfrac{3}{10} = \dfrac{\cancel{10}}{3\cancel{0}} = \dfrac{3}{5}$

$= \dfrac{3 \times 9}{3 \times 10}$

$\dfrac{3}{4} \div \dfrac{2}{3} = \dfrac{3}{4} \times \dfrac{3}{2} = \dfrac{9}{8}$

The method for multiplying and dividing fractions also works with rational expressions. Remember that to multiply two fractions, you multiply the numerators and multiply the denominators. To divide two fractions, you multiply by the multiplicative inverse, or the reciprocal, of the divisor.

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Multiplication

$$\frac{2}{9} \cdot \frac{15}{4} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 5}{\cancel{3} \cdot 3 \cdot \underset{1}{\cancel{2}} \cdot 2} = \frac{5}{3 \cdot 2} = \frac{5}{6}$$

Division

$$\frac{3}{5} \div \frac{6}{35} = \frac{3}{5} \cdot \frac{35}{6} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot 7}{\cancel{5} \cdot 2 \cdot \underset{1}{\cancel{3}}} = \frac{7}{2}$$

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State any restrictions on the following rational expression and simplify:

$$\left(\frac{2x^3 + 10x^2 + 12x}{2x^2 + 14x}\right)\left(\frac{x+7}{x^2-9}\right)$$

$$\left[\frac{2x(x+3)(x+2)}{2x(x+7)}\right]\cdot\left[\frac{x+7}{(x-3)(x+3)}\right]$$

$$\frac{2x(x+3)(x+2)(x+7)}{2x(x+7)(x-3)(x+3)}$$

$$= \frac{x+2}{x-3}, \; x \neq 0, 3, -3, -7$$

State any restrictions on the following rational expression and simplify:

$$\frac{x+7}{x^2-9} \div \frac{x^2+9x+14}{3x^2-9x}$$

$$\frac{x+7}{(x-3)(x+3)} \cdot \frac{3x(x-3)}{(x+7)(x+2)}$$

Must Consider ALL factors for restrictions

$$= \frac{3x}{(x+3)(x+2)} \quad , x \neq \pm 3, 0, -7, -2$$

Practice Problems...

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