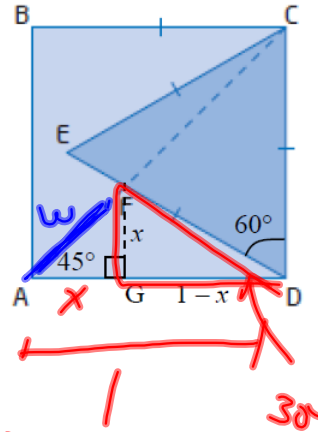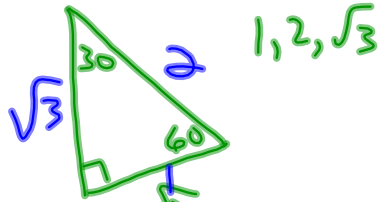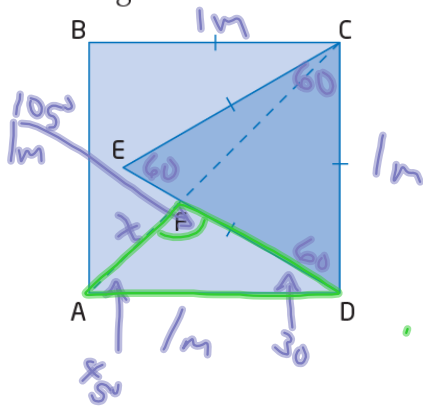Let's revisit the bonus problem...

A square, ABCD, has a perimeter of 4 m.
△CDE is an equilateral triangle inside
the square. The intersection of AC and
DE occurs at point F. What is the exact
length of AF?

B ──1m── C

E

A ──1m── D

1, 2, √3

30
√3
60
2

$$\frac{(\sin 30°)}{\sin 105°} = \frac{x}{\sin 30°} (\cancel{\sin 30°})$$

$$\boxed{x = \frac{\sin 30°}{\sin 105°}}$$

B ──── C

E

45° 60°

A x G 1−x D

30

$$\tan 30° = \frac{x}{1-x}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{1-x}$$

$$x\sqrt{3} = 1-x$$

$$x\sqrt{3} + x = 1$$

$$\frac{x(\sqrt{3}+1)}{\sqrt{3}+1} = \frac{1}{\sqrt{3}+1}$$

$$x = \frac{1}{\sqrt{3}+1}$$

$$\left(\frac{1}{\sqrt{3}+1}\right)^2 + \left(\frac{1}{\sqrt{3}+1}\right)^2 = w^2$$

Dividing Radical Expressions

Property: $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$ if $b \neq 0$

Let's look at a few examples...

$\dfrac{\sqrt{15}}{\sqrt{3}} = \sqrt{\dfrac{15}{3}} = \sqrt{5}$

$\dfrac{24\sqrt{48}}{3\sqrt{12}}$ $= 8\sqrt{4}$
$= 16$

$= \dfrac{24(4\sqrt{3})}{3(2\sqrt{3})}$

$= \dfrac{96\sqrt{3}}{6\sqrt{3}}$

$= 16$

$\dfrac{7\sqrt[3]{320}}{14\sqrt[3]{10}}$

$= \dfrac{1}{2}\sqrt[3]{32}$

$= \dfrac{1}{2}(2\sqrt[3]{4})$

$= \sqrt[3]{4}$

$\dfrac{-8x^5\sqrt{48x^{11}}}{3x^2\sqrt{2x^3}}$

$= \dfrac{-8}{3}x^3\sqrt{24x^8}$

$= \dfrac{-8}{3}x^3\left(\dfrac{2x^4\sqrt{6}}{1}\right)$

$= \dfrac{-16x^7\sqrt{6}}{3}$

What if the everything does not divide evenly??

$$\frac{\sqrt{10}}{\sqrt{6}} = ??$$

Rationalizing the Denominator

$$= 1$$

rationalize

- convert to a rational number without changing the value of the expression

- If the radical is in the denominator, both the numerator and denominator must be multiplied by a quantity that will produce a rational denominator.

$$\frac{\sqrt{10}}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right) = \frac{\sqrt{60}}{6} = \frac{2\sqrt{15}}{6}$$

$$= \frac{\sqrt{15}}{3}$$

Student B

$$\frac{\sqrt{5}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{15}}{3}$$

3

$$\frac{6}{\sqrt{12}}\left(\frac{\sqrt{12}}{\sqrt{12}}\right) \qquad \frac{6}{2\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) \qquad \frac{\sqrt{28}}{\sqrt{8}}\left(\frac{\sqrt{8}}{\sqrt{8}}\right) = \frac{\sqrt{224}}{8}$$

$$= \frac{6\sqrt{12}}{12} \qquad\qquad = \frac{6\sqrt{3}}{2(3)} \qquad\qquad \frac{\sqrt{7}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{4\sqrt{14}}{8}$$

$$= \frac{6(2\sqrt{3})}{12} = \frac{12\sqrt{3}}{12} \qquad = \frac{6\sqrt{3}}{6} \qquad = \frac{\sqrt{14}}{2} \qquad\qquad = \frac{1}{2}\sqrt{14}$$

$$\boxed{= \sqrt{3}} \qquad\qquad = \sqrt{3} \qquad \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$$

$$\frac{12\sqrt{18}}{8\sqrt{8}}\left(\frac{\sqrt{8}}{\sqrt{8}}\right) = \frac{12\sqrt{9}}{8\sqrt{4}} \qquad\qquad \frac{-3\sqrt{8a}}{a\sqrt{5a^3}}\left(\frac{\sqrt{5a^3}}{\sqrt{5a^3}}\right)$$

$$= \frac{36\sqrt{2}}{16\sqrt{2}} \quad\Big\{\quad \frac{12\sqrt{144}}{8(8)} \quad = \frac{12(3)}{8(2)} \qquad = \frac{-3\sqrt{40a^4}}{a(5a^3)}$$

$$= \frac{36}{16} \qquad\qquad = \frac{12(12)}{64} \qquad = \frac{36}{16} \qquad\qquad = \frac{-6a^2\sqrt{10}}{5a^4}$$

$$\boxed{= \frac{9}{4}} \qquad\qquad = \frac{144}{64} \qquad = \frac{9}{4}$$

$$\boxed{= \frac{9}{4}} \qquad\qquad = \frac{-6}{5}a^2\sqrt{10}$$

$$\text{OR}$$

$$= \frac{-6\sqrt{10}}{5a^2}$$

$$\sqrt{6}\cdot\sqrt{6} = 6$$

$$6^{1/2}\cdot 6^{1/2} \qquad\qquad = \frac{-6}{5a^2}\sqrt{10}$$

$$6^{2/2} = 6$$

What about rationalizing with other indicies??

$$6^{1/3} \cdot 6^{1/3} \cdot 6^{1/3}$$

$$6^{3/3} = 6^1$$

$$\frac{5\sqrt[3]{4w^2}}{w\sqrt[5]{8}} \qquad \left(\frac{\sqrt[5]{8}}{\sqrt[5]{8}}\right)^4$$

$$\frac{9\sqrt{24}}{\sqrt[3]{6}} \qquad \left(\frac{\sqrt{6}}{(\sqrt[3]{6})^2}\right)^2$$

$$\frac{9\sqrt{6}(\sqrt[3]{6})^2}{6}$$

$$= 3\sqrt{6}\,(\sqrt[3]{6})^2$$

$$= 3(6)^{1/2}(6)^{2/3}$$

$$= 3(6)^{3/6}(6)^{4/6}$$

$$= 3 \cdot (6)^{7/6}$$

$$= 3\left(\sqrt[6]{6}\right)^7$$