$$|-8| = 8$$
$$|8| = 8$$
$$|x| = 8$$
$$x = \pm 8$$

Write without Absolute Value Bars

$$|-8| = 8$$

Between Bars Negative

$$-(-8) = 8$$

$$|8| = 8$$

Between Bars Positive

Drop Bars

$$8 = 8$$

Here is a more Mathematical definition...

ABSOLUTE VALUE

E.1 DEFINITION. The ***absolute value*** or ***magnitude*** of a real number a is denoted by $|a|$ and is defined by

$$|a| = \begin{cases} a & \text{if} \quad a \geq 0 \\ -a & \text{if} \quad a < 0 \end{cases}$$

BDP

BDN

Piecewise function

Example 1

$$|5| = 5 \qquad \left|-\tfrac{4}{7}\right| = -\left(-\tfrac{4}{7}\right) = \tfrac{4}{7} \qquad |0| = 0$$

Since $5 > 0$ | Since $-\frac{4}{7} < 0$ | Since $0 \geq 0$

Note that the effect of taking the absolute value of a number is to strip away the minus sign if the number is negative and to leave the number unchanged if it is nonnegative.

Expressing without absolute value symbol...

Example:

$$|x+3| \longrightarrow \begin{cases} x+3 & , \text{if } x+3 \geqq 0 \quad \text{BBP} \\ -(x+3) & , \text{if } x+3<0 \end{cases}$$

$$\longrightarrow \begin{cases} x+3, & \text{if } x \geqq -3 \quad \text{BBN} \\ -x-3, & \text{if } x<-3 \end{cases}$$

$$|x-5| \longrightarrow \begin{cases} x-5, & \text{if } x-5 \geqq 0 \\ -(x-5), & \text{if } x-5<0 \end{cases}$$

$$\begin{cases} x-5, & \text{if } x \geqq 5 \\ -x+5, & \text{if } x<5 \end{cases}$$

$|5x+4| \longrightarrow \begin{cases} 5x+4, & \text{if } 5x+4 \geqq 0 \\ -(5x+4), & \text{if } 5x+4<0 \end{cases}$

$\longrightarrow \begin{cases} 5x+4, & \text{if } x \geqq -\frac{4}{5} \\ -5x-4, & \text{if } x < -\frac{4}{5} \end{cases}$

$|3-4x| \longrightarrow \begin{cases} 3-4x, & \text{if } x \leqq \frac{3}{4} \\ -3+4x, & \text{if } x > \frac{3}{4} \end{cases}$

$3-4x \geqq 0$

$-4x \geqq -3$

$x \leqq \frac{3}{4}$

Practice Problems...

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#6, 7 c,d , 11, 12, 14, 15, 18, 24

CHAPTER 6

Rational Expressions and Equations

In this chapter, you will learn about the algebra of rational expressions and equations. Compare the skills you learn in the chapter with those you learned in the arithmetic of fractions. They are very similar.

Not a Rational Expression $\frac{\sqrt{x+3}}{3x^2}$

Definition

A **rational expression** (or **algebraic fraction)** is a fraction with a polynomial in the numerator and and a nonzero polynomial in the denominator.

$$\text{e.g.}\quad \frac{-4}{x},\qquad \frac{2y^3-4}{-5y+9},\qquad \frac{-4a^3-7}{a^2-3a},\qquad \frac{x-4}{x^2+3x-28}$$

Definition

The **domain of a rational expression (algebraic fraction)** is the set of all real numbers except the value(s) of the variable that result in division by zero when substituted into the expression. To find the values of the variable *excluded* from the domain, set the denominator equal to zero and solve.

non-permissible value

- any value for a variable that makes an expression undefined
- in a rational expression, a value that results in a denominator of zero
- in $\frac{x+2}{x-3}$, you must exclude the value for which $x - 3 = 0$, which is $x = 3$

Examples...

$$\frac{x+2}{3x-5}$$

$3x-5=0$

$3x=5$

$x=\frac{5}{3}$

Domain:

$\{x \in R, x \neq \frac{5}{3}\}$

$$\frac{3x-5}{x^2-13x+30}$$

$\{x \in R, x \neq 3 \text{ or } 10\}$

$x^2-13x+30=0$

$(x-3)(x-10)=0$

$x=3, 10$

Example 1

Find the values excluded from domain of each algebraic fraction above.

a. $\dfrac{-4}{x}$

$\{x \in R, x \neq 0\}$

$-5y+9=0$

$-5y=-9$

$y=\frac{9}{5}$

b. $\dfrac{2y^3-4}{-5y+9}$

$\{y \in R, y \neq \frac{9}{5}\}$

c. $\dfrac{-4a^3-7}{a^2-3a}$

$a^2-3a=0$

$a(a-3)=0$

$a=0, 3$

$\{a \in R, a \neq 0 \text{ or } 3\}$

d. $\dfrac{x-4}{x^2+3x-28}$

$x^2+3x-28=0$

$(x+7)(x-4)=0$

$x=-7, 4$

$\{x \in R, x \neq -7 \text{ or } 4\}$

Chapter 6

Non-Permissible Values

For each rational expression, determine all non-permissible values.
State the domain for each expression. Use the pen tool to write your answers.
Drag the Answer tab under each question to reveal the answer.

Answer

1. $\dfrac{7x}{8x+16}$

2. $\dfrac{p^2-25}{p^2-10p-24}$

3. $\dfrac{3}{x^3}$

Hint

4. $\dfrac{2x-y}{xy}$

$x, y \in R, x \neq 0, y \neq 0\}$

5. $\dfrac{2x^2+5y}{x-y}$

$\{x, y \in R, x \neq y\}$

6. $\dfrac{2x^2}{x^2-4}$

$(x-2)(x+2)$
$x \neq 2, -2$

Simplifying Rational Expressions

Definition

To **simplify a rational expression (an algebraic fraction)** means to write the fraction so that there are no common factors other than 1 or -1.

Steps to simplify an algebraic fraction.

1. Factor the numerator and denominator.
2. Divide out all common factors.

In other words we are reducing algebraic fractions...

- Reduce the following fraction...

$$\frac{8}{10}=\frac{4\times\cancel{2}}{5\times\cancel{2}}=\frac{4}{5}$$

- Similar process with rational expressions...

$$\frac{5x^2-20x}{10x^2+20x}=\frac{5\cancel{x}(x-4)}{10\cancel{x}(x+2)}$$

$$=\frac{1(x-4)}{2(x+2)}$$

$$=\frac{x-4}{2(x+2)}$$

(*Note* Monomials are already in factored form)

1. Simplify $\frac{4\cancel{x^3}y^{\cancel{3}}}{6x^4\cancel{y^2}}$

$= \frac{2y^3}{3x}$

$= \frac{2}{3}x^{-1}y^3$

No Negative exponents

$= \frac{2y^3}{3x}$

2. Simplify $\frac{6x^5y^1}{12x^2y^3}$

$= \frac{1x^3}{2y^2} = \frac{x^3}{2y^2}$

3. Simplify $\frac{x^2-4}{x^2-2x-8}$

$$\frac{(x-2)\cancel{(x+2)}}{(x-4)\cancel{(x+2)}}$$

$$= \frac{x-2}{x-4}$$

4. Simplify $\frac{\cancel{x-4}}{\cancel{4-x}}$

$$\frac{7-3}{3-7} = \frac{4}{-4}$$

$$\frac{-1\cancel{(-x+4)}}{\cancel{4-x}}$$

$$= -1$$

$$\frac{\cancel{x+3}}{\cancel{3+x}} = 1$$

Example 3

1. Simplify $\dfrac{\overset{-1}{\cancel{(3-4x)}}(x-7)}{(x+5)\cancel{(4x-3)}}$

$$\dfrac{-(x-7)}{x+5}$$

$$= \dfrac{-x+7}{x+5}$$

- Common factor
- Simple Trinomial
- Decomposition
- Diff. of Squares

2. Simplify $\dfrac{14-7x}{x^3-2x^2}$

$$\dfrac{7\overset{-1}{\cancel{(2-x)}}}{x^2\cancel{(x-2)}}$$

$$= \dfrac{-7}{x^2}$$

Simplify $\dfrac{9a-3}{3a^2+11a-4}$ -12

$9a-3$

$3(3a-1)$

$3a^2+11a-4$

$3a^2+12a-1a-4$

$3a(a+4)-1(a+4)$

$(a+4)(3a-1)$

$$= \dfrac{3\overset{1}{\cancel{(3a-1)}}}{(a+4)\cancel{(3a-1)}}$$

$$= \dfrac{3}{a+4}$$

Simplify $\dfrac{x^2+8x+7}{x^2-4x-5}$

$$\dfrac{(x+7)\cancel{(x+1)}}{(x-5)\cancel{(x+1)}}$$

$$= \dfrac{x+7}{x-5}$$