**Set C** *Dawson College: Winter 2011*

**1.** Find each indefinite integral.

a) $\displaystyle\int \frac{3x^2+4x+4}{x^3+x}\,dx$

b) $\displaystyle\int \tan^3 x\,dx$

d) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

e) $\displaystyle\int \frac{dx}{2+\sqrt{x}}$

**2.** Evaluate the definite integral $\displaystyle\int_0^{\pi} \sin^2(3\theta)\,d\theta$.

**1.** Find each indefinite integral.

a) $\displaystyle\int \frac{3x^2+4x+4}{x^3+x}\,dx$

$$\int \frac{3x^2+4x+4}{x(x^2+1)}\,dx$$

$$\frac{A}{x}+\frac{Bx+C}{x^2+1}=\frac{3x^2+4x+4}{x(x^2+1)}$$

$$A(x^2+1)+(Bx+C)x=3x^2+4x+4$$

$$Ax^2+A+Bx^2+Cx=3x^2+4x+4$$

$A+B=3 \qquad A=4 \qquad C=4$

$4+B=3$

$B=-1$

$$\int \frac{4}{x}\,dx+\int \frac{-x+4}{x^2+1}\,dx$$

$$\int \frac{4}{x}\,dx-\frac{1}{2}\int \frac{2x}{x^2+1}\,dx+\int \frac{4}{x^2+1}\,dx$$

$$=4\ln|x|-\frac{1}{2}\ln(x^2+1)+4\tan^{-1}x+C$$

b) $\int \tan^3 x \, dx$

$$\int \tan^2 x \tan x \, dx$$

$$\int (\sec^2 x - 1) \tan x \, dx$$

$$\int \left(\tan x \sec^2 x - \tan x\right) dx$$

$$\int \left[(\tan x)' \sec^2 x - \frac{\sin x}{\cos x}\right] dx$$

$$= \frac{1}{2} \tan^2 x - \ln|\cos x| + C$$

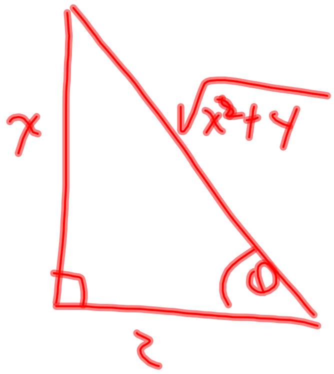
c) $\int x^2 e^{4x}\,\mathrm{d}x$

| | | |
|---|---|---|
| $x^2$ | $+$ | $e^{4x}$ $(4)$ |
| $2x$ | $-$ | $\frac{1}{4}e^{4x}$ |
| $2$ | $+$ | $\frac{1}{16}e^{4x}$ |
| $0$ | | $\frac{1}{64}e^{4x}$ |

OR By Parts Twice!!

$$= \frac{x^2 e^{4x}}{4} - \frac{xe^{4x}}{8} + \frac{e^{4x}}{32} + C$$

d) $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$




$$2\sec\theta = \sqrt{x^2+4}$$

$$(2\tan\theta)^2 = (x)^2 = 4\tan^2\theta = x^2$$

$$2\sec^2\theta\, d\theta = dx$$

$$\int \frac{2\sec^{\cancel{2}}\theta\, d\theta}{4\tan^2\theta\,(2\cancel{\sec\theta})}$$

$$\frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$\frac{1}{4}\int \cot^2\theta\left(\frac{1}{\cos\theta}\right)d\theta$$

$$\frac{1}{4}\int \frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\left(\frac{1}{\cancel{\cos\theta}}\right)d\theta$$

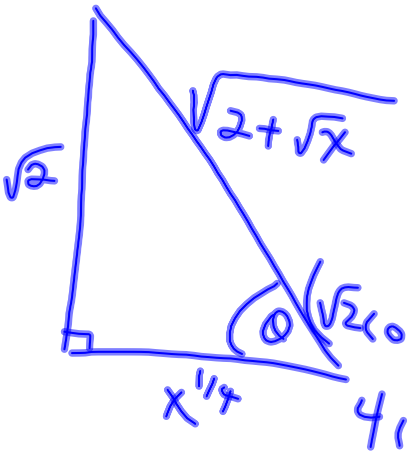
$$\frac{1}{4}\int (\sin\theta)^{-2}\cos\theta\, d\theta$$

$$= -\frac{1}{4}(\sin\theta)^{-1} + C$$

$$= -\frac{1}{4}\left(\frac{x}{\sqrt{x^2+4}}\right)^{-1} + C$$

$$= -\frac{1}{4}\left(\frac{\sqrt{x^2+4}}{x}\right) + C$$

e) $\int \frac{dx}{2+\sqrt{x}}$




$(\sqrt{2}\cot\theta)^4 = (x^{\frac{1}{4}})^4$

$4\cot^4\theta = x$

$16(\cot\theta)^3(-\csc^2\theta\, d\theta) = dx$

$\sqrt{2}\csc\theta = \sqrt{2+\sqrt{x}}$

$2\csc^2\theta = 2+\sqrt{x}$

$$\int \frac{-16\cot^3\theta \cancel{\csc^2\theta}\, d\theta}{2\cancel{\csc^2\theta}}$$

$$-8\int \cot^3\theta\, d\theta$$

$$-8\int \cot^2\theta \cot\theta\, d\theta$$

$$-8\int (\csc^2\theta - 1)\cot\theta\, d\theta$$

$$+8\int (\cot\theta)^1 \underset{(-)}{\csc^2\theta}\, d\theta + 8\int \frac{\cos\theta}{\sin\theta}\, d\theta$$

$$= 4\cot^2\theta + 8\ln|\sin\theta| + C$$

$$= 4\left(\frac{\sqrt[4]{x}}{\sqrt{2}}\right)^2 + 8\ln\left(\frac{\sqrt{2}}{\sqrt{2+\sqrt{x}}}\right) + C$$

$$= 2\sqrt{x} + 8\ln\left(\frac{\sqrt{2}}{\sqrt{2+\sqrt{x}}}\right) + C$$

**2.** Evaluate the definite integral $\int_0^\pi \sin^2(3\theta)\, d\theta$.

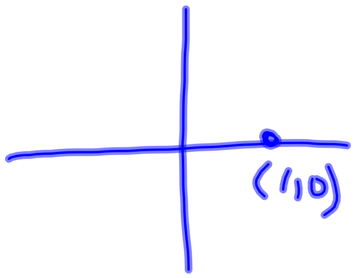
$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos(6\theta) = 1 - 2\sin^2(3\theta)$$

$$\sin^2 3\theta = \frac{1 - \cos 6\theta}{2}$$

$$\int_0^\pi \frac{1}{2}\, d\theta - \frac{1}{6}\cdot\frac{1}{2}\int_0^\pi \cos 6\theta\,(6)\, d\theta$$

$$= \frac{\theta}{2} - \frac{1}{12}\sin(6\theta)\Big|_0^\pi$$




$$= \left[\frac{\pi}{2} - \frac{1}{2}\sin(6\pi)\right] - \left[0 - \frac{1}{12}\sin(0)\right]$$

$$= \left(\frac{\pi}{2} - 0\right) - 0$$

$$= \frac{\pi}{2}$$

Middlebury College 2012

Evaluate each of the following integrals. If there is a particular technique that you use, name it.

(a) $\int x \sec^2(x)\, dx$

(b) $\int \tan^2 x \sec^4 x\, dx$

(d) $\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\, dx$

(c) $\int_0^1 \frac{1}{(x^2+1)^2}\, dx$ (Hint: use a trigonometric substitution.)