Warm-Up...

$\int \sin^3 x \, dx$

$\int \sin x (\sin^2 x) dx$

$\int \sin x (1 - \cos^2 x) dx$

$\int (\sin x - \sin x \cos^2 x) dx$ (du, u^n)

$\int \sin x dx + \int (\cos x)^2 \cdot -\sin x dx$

$-\cos x + \frac{1}{3}\cos^3 x + C$

$\int tan^4 x \, dx$

$\int (\tan^2 x) \tan^2 x \, dx$

$\int (\sec^2 x - 1) \tan^2 x dx$

$\int \sec^2 x \tan^2 x dx - \int \tan^2 x dx$ (du, u^n)

$\int (\tan x)^2 \sec^2 x dx - \int (\sec^2 x - 1) dx$

$\int (\tan x)^2 \sec^2 x dx - \int \sec^2 x dx - \int dx$

$= \frac{1}{3}\tan^3 x - \tan x - x + C$

$\int \tan^3 x \sec^4 x \, dx$ — Peel off a $\tan x$

$\int (\tan x) \tan^2 x \sec^4 x dx$

$\int \tan x (\sec^2 x - 1) \sec^4 x dx$

$\int (\sec^6 x \tan x - \sec^4 x \tan x) dx$

$\int \sec^5 x (\sec x \tan x) dx - \int \sec^3 x (\sec x \tan x) dx$

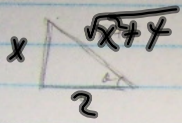
$= \frac{1}{6}\sec^6 x - \frac{1}{4}\sec^4 x + C$

$\int (\sec^4 x \tan x - \sec^6 x \tan x)\,dx$

$\int (\sec x)^3 \sec x \tan x\,dx - \int (\sec x)^5 \sec x \tan x\,dx$

$\frac{\sec^4 x}{4} - \frac{\sec^6 x}{6} + c$

4) $\int \frac{x+4}{x^2+4}\,dx$



$\tan\theta = \frac{x}{2}$ $\qquad \sec\theta = \frac{\sqrt{x^2+4}}{2}$ $\qquad \theta = \tan^{-1}\frac{x}{2}$

$x = 2\tan\theta$ $\qquad 4\sec^2\theta = x^2+4$ $\qquad \cos\theta = \frac{4}{\sqrt{x^2+4}}$

$dx = 2\sec^2\theta\,d\theta$

$\int \frac{(4\tan\theta + 4)(4\sec^2\theta\,d\theta)}{16\sec^2\theta}$

$\int \tan\theta + 1\,d\theta$

$\int \frac{\sin\theta}{\cos\theta}\,d\theta + \int d\theta$

$-\ln|\cos\theta| + \theta$

$\int \frac{(2\tan\theta + 4)\,2\sec^2\theta\,d\theta}{4\sec^2\theta}$

$\int \frac{4(\tan\theta + 2)}{4}\,d\theta$

$\int \tan\theta\,d\theta + \int 2\,d\theta$

$-\int \frac{-\sin\theta}{\cos\theta}\,d\theta + 2\int d\theta$

$= \ln|\cos\theta| + 2\theta$

$= -\ln\left(\frac{2}{\sqrt{x^2+4}}\right) + 2\tan^{-1}\left(\frac{x}{2}\right) + C$

$$\int \frac{x+4}{x^2+4}\,dx$$

$$\frac{1}{2}\int \frac{(2)x}{x^2+4}\,dx + \int \frac{4}{x^2+4}\,dx$$

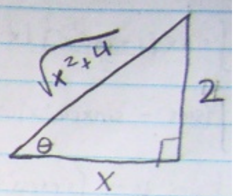
$$\frac{1}{2}\ln(x^2+4) + \int \sin^2\theta\,(-2\csc^2\theta\,d\theta)$$

$$\frac{\ln(x^2+4)}{2} + \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) - \int 2\,d\theta$$

$$\frac{\ln(x^2+4)}{2} + \frac{1}{2}x - \int \frac{\cos 2x}{2}$$

$$\frac{1}{2}\int \cos 2x$$

$$\frac{\ln(x^2+4)}{2} + \frac{x}{2} - \frac{1}{4}\sin 2x$$



$$\sin\theta = \frac{2}{\sqrt{x^2+4}}$$

$$\sin^2\theta = \frac{4}{x^2+4}$$

$$\cot\theta = \frac{x}{2}$$

$$2\cot\theta = x$$

$$-2\csc^2\theta\,d\theta = dx$$

$$\frac{1}{2}\ln(x^2+4) - 2\theta + C$$

$$= \frac{1}{2}\ln(x^2+4) - 2\cot^{-1}\left(\frac{x}{2}\right) + C$$

$$\int (\sec^2 x - 1)^2 \, dx$$

$$\int \sec^4 x \, dx - 2\sec^2 x + 1$$

$$\int (1 + \tan^2 x) \sec^2 x \, dx - 2\int \sec^2 x \, dx + \int dx$$

Here is another tricky one involving trigonometry....

$$\int \sec x dx =$$

$$\sec x = \frac{1}{\cos x} = \frac{\cos x}{\cos^2 x} = \frac{\cos x}{1 - \sin^2 x} .$$

ry the substitution $u = \sin x,\ du = \cos x dx$. Then

$$\int \sec x dx = \int \frac{du}{1 - u^2} .$$

e a dead end, but a little algebra pulls us through. The identity

$$\frac{1}{1 - u^2} = \frac{1}{2}\left(\frac{1}{1 + u} + \frac{1}{1 - u}\right)$$

$$\int \frac{du}{1 - u^2} dx = \frac{1}{2} \int \left(\frac{1}{1 + u} + \frac{1}{1 - u}\right) du = \frac{1}{2} (\ln (1 + u) - \ln (1 - u)) + C$$

Here is another neat trick...

$$\int \sec x dx =$$

Hint 1...here is what the answer will look like with this trick

$$\ln|\sec\theta + \tan\theta| + C$$

Hint 2...It is always legal to multiply by 1

$$\int \sec x \left(\frac{\sec x + \tan x}{\sec x + \tan x}\right) dx$$

$$\int \frac{\sec^2 x + \sec x \tan x}{\tan x + \sec x} dx \Leftarrow \frac{du}{u}$$

$$\ln|\tan x + \sec x| + C$$

Attachments

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc