Practice Problems...

Evaluate each of the following:

$$\int \frac{1}{(16-x^2)^{\frac{3}{2}}}\,dx$$ (UNB: 2005)

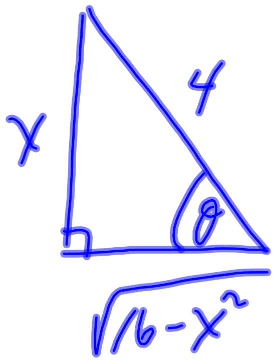
$\to \left(\sqrt{16-x^2}\right)^3$

$$\int \frac{\sqrt{2-x^2}}{x}\,dx$$ (UNB: 2004)

$$\int \left(\csc^2 t \cot^2 t\right)\,dt$$ (UNB: 2004)

$$\int \frac{\ln(\ln x)}{x}\,dx$$ (StFX: 2005)

$$\int \frac{1}{(16-x^2)^{\frac{3}{2}}}\,dx$$ (UNB: 2005)



$\sin\theta = \frac{x}{4}$

$x = 4\sin\theta$

$dx = 4\cos\theta\,d\theta$

$\cos\theta = \frac{\sqrt{16-x^2}}{4}$

$4\cos\theta = \sqrt{16-x^2}$

$$\int \frac{4\cos\theta\,d\theta}{(4\cos\theta)^3}$$

$$\frac{1}{16}\int \frac{\cos\theta}{\cos^3\theta}\,d\theta$$

$$\frac{1}{16}\int \frac{1}{\cos^2\theta}\,d\theta$$

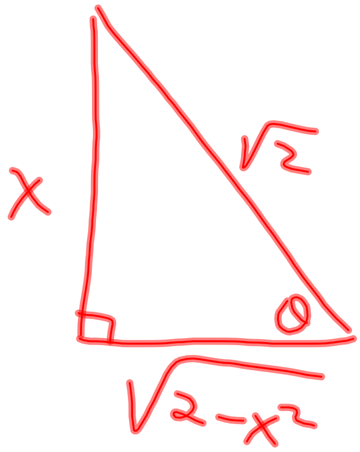
$$\frac{1}{16}\int \sec^2\theta\,d\theta$$

$d(\tan\theta) = \sec^2\theta$

$$\frac{1}{16}\underbrace{\tan\theta}_{\downarrow} + C$$

$$= \frac{1}{16}\left(\frac{x}{\sqrt{16-x^2}}\right) + C$$

$$\int \frac{\sqrt{2-x^2}}{x}\,dx$$ (UNB: 2004)



$\sin\theta = \frac{x}{\sqrt{2}}$

$\sqrt{2}\sin\theta = x$ $\qquad \sqrt{2}\cos\theta\,d\theta = dx$

$\cos\theta = \frac{\sqrt{2-x^2}}{\sqrt{2}}$

$\sqrt{2}\cos\theta = \sqrt{2-x^2}$

$$\int \frac{(\cancel{\sqrt{2}}\cos\theta)\sqrt{2}\cos\theta\,d\theta}{\cancel{\sqrt{2}}\sin\theta}$$

$$\sqrt{2}\int \frac{\cos^2\theta\,d\theta}{\sin\theta}$$

$$\sqrt{2}\int \frac{1-\sin^2\theta}{\sin\theta}\,d\theta$$

$$\sqrt{2}\int\left(\frac{1}{\sin\theta}-\frac{\sin^2\theta}{\sin\theta}\right)d\theta$$

$$\sqrt{2}\int \csc\theta\,d\theta - \sqrt{2}\int \sin\theta\,d\theta$$

$$\sqrt{2}\int \csc\theta\,d\theta\left(\frac{\csc\theta+\cot\theta}{\csc\theta+\cot\theta}\right)+\sqrt{2}\cos\theta + C$$

$$-\sqrt{2}\int \frac{(\csc^2\theta+\csc\theta\cot\theta)^{-1}}{\cot\theta+\csc\theta}\,d\theta \quad +\sqrt{2}\cos\theta + C$$

$$-\sqrt{2}\ln\left|\cot\theta+\csc\theta\right| + \sqrt{2}\cos\theta + C$$

$$-\sqrt{2}\ln\left|\frac{\sqrt{2-x^2}}{x}+\frac{\sqrt{2}}{x}\right| + \cancel{\sqrt{2}}\left(\frac{\sqrt{2-x^2}}{\cancel{\sqrt{2}}}\right)+C$$

$$-\int -(\overset{du}{\csc^2 t}\;\overset{u^n}{\cot^2 t})\ dt$$ (UNB: 2004)

$$= -\frac{1}{3}\cot^3 t + C$$

$$\int \frac{\ln(\ln x)}{x}\,dx$$ (StFX: 2005)

$$\left.\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}dx \end{aligned}\right\} \int \ln u \, du$$

$$\int u\,dv = uv - \int v\,du$$

$w = \ln u \qquad dv = du$

$dw = \frac{1}{u}du \qquad v = u$

avoid confusion

$$\boxed{\int w\,dv = wv - \int v\,dw}$$

$$= u\ln u - \int \cancel{u}\frac{1}{\cancel{u}}du$$

$$= u\ln u - \int du$$

$$= u\ln u - u + C$$

$$= \ln x \ln(\ln x) - \ln x + C$$

Sometimes completing the square can be a useful strategy to evaluate an integral...

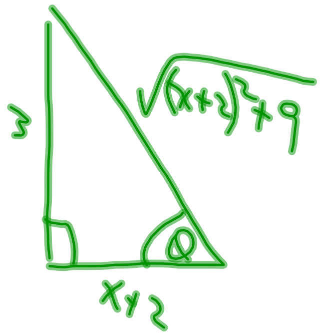
$$\int \frac{1}{x^2+4x+13}\,dx$$

$x^2+4x+13$

$(x^2+4x+4)+13-4$

$(x+2)^2+9$

$\int \frac{dx}{(x+2)^2+9} \Rightarrow$



$\int \frac{-3\cancel{\csc^2\theta}\,d\theta}{9\cancel{\csc^2\theta}}$

$-\frac{1}{3}\int d\theta$

$= -\frac{1}{3}\theta + C$

$= -\frac{1}{3}\cot^{-1}\left(\frac{x+2}{3}\right) + C$

$\cot\theta = \frac{x+2}{3}$

$3\cot\theta - 2 = x$

$-3\csc^2\theta\,d\theta = dx$

$3\csc\theta = \sqrt{(x+2)^2+9}$

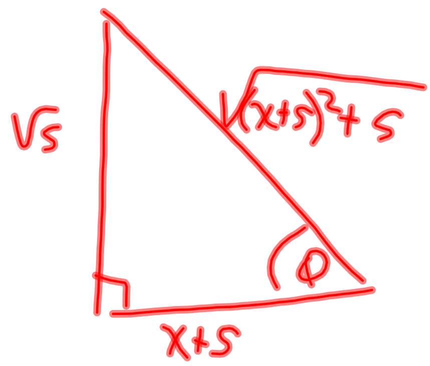
$9\csc^2\theta = (x+2)^2+9$

$$\int \frac{dx}{x^2 + 10x + 30}$$

$$(x^2+10x+25)+30-25$$

$$(x+5)^2+5$$

$$\int \frac{dx}{(x+5)^2+5}$$



$$\sqrt{5}\cot\theta = x+5 \qquad \sqrt{5}\csc\theta = \sqrt{(x+5)^2+5}$$

$$-\sqrt{5}\csc^2\theta\, d\theta = dx \qquad 5\csc^2\theta = (x+5)^2+5$$

$$\int \frac{-\sqrt{5}\cancel{\csc^2\theta}\, d\theta}{5\cancel{\csc^2\theta}}$$

$$-\frac{\sqrt{5}}{5}\int d\theta$$

$$-\frac{\sqrt{5}}{5}\theta + C$$

$$-\frac{\sqrt{5}}{5}\cot^{-1}\left(\frac{x+5}{\sqrt{5}}\right) + C$$

Attachments

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc